ETH Zürich FS 2013
D-MATH
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Applied Stochastic Processes

Exercise Sheet 4

Please hand in by 13:00 on Friday 26-Apr-2013 in the assistant's box in HG E 65.

Exercise 4-1

Suppose that $f_x(t)$ is a risk process with starting capital x, as on page 45 of the lecture notes. The claim amounts X_i are independent, with common distribution function G. Define the no-ruin probability starting with capital x as

$$R(x) = P[f_x(t) \geq 0 \text{ for all } t > 0].$$

Show that if $\lambda E[X_1] < c$, then R' satisfies the renewal equation with defect

$$R'(t) = F'(t)R(0) + \int_0^t R'(t-s)\,\mathrm{d}F(s), \tag{$*$}$$

where $F(t) = \int_0^t \frac{\lambda}{c} P[X_1 > u]\,\mathrm{d}u$ for $t \geq 0$.

Exercise 4-2

Suppose that $\lambda E[X_1] < c$. Use the renewal equation $(*)$ from Exercise 4-1 to show

a) $R(\infty) = 1$,

b) $R(0) = 1 - \frac{\lambda}{c} E[X_1]$,

c) $R(t) = R(0)(1 + M(t))$,

where $M(t)$ is the defective renewal function corresponding to F.
Hint: To show b) and c), solve the Laplace transform version of the renewal equation $(*)$.

Exercise 4-3

Using the results from Exercise 4-2 b), compute $R(x)$ explicitly in the case when the claim sizes X_i are exponentially distributed with $E[X_1] = \mu < c/\lambda$.

Exercise 4-4

Show that a renewal process with renewal function $M(t) = ct$, $t \geq 0$ for some constant $c > 0$ is a Poisson process. *Hint:* The Laplace transform determines the distribution.

Exercise 4-5

Suppose that the lifetimes of individuals of a population are independent random variables with a non-arithmetic distribution F. At the time of death, an individual produces k successors with probability $p_k \geq 0$. We assume $\sum_{k=0}^{\infty} p_k = 1$ and $\sum_{k=0}^{\infty} kp_k = m > 1$. Furthermore, suppose that number of produced offspring is independent of the lifetimes, and between individuals. At time 0, there is one individual with age 0.

Let X_t be the size of the population at time t and $M(t) = E[X_t]$.

a) Show that $M(t)$ fulfills an (improper) renewal equation.

b) Determine the asymptotic behaviour of $M(t)$ in dependence on F.

There are office hours held on Mondays and Thursdays at 12-13 in HG G 32.6.