

Applied Stochastic Processes

Exercise Sheet 7

*You can hand in by 13:00 on Friday 31-May-2013 in the assistant's box in HG E 65.
Corrected works will be returned there on the following Tuesday.*

Exercise 7-1

Kolmogorov equations. Consider a pure jump process with generator matrix

$$\Lambda = \frac{1}{4} \begin{pmatrix} -8 & 3 & 5 \\ 6 & -8 & 2 \\ 2 & 2 & -4 \end{pmatrix}.$$

Compute the transition probability matrix $r(t)$.

Note: You can use computer software for matrix operations.

Exercise 7-2

Memoryless property of Exp. A company owns M identical machines, which are maintained by one service person. The probability that a given machine fails in a small time interval of length h is $\mu h + o(h)$, independently of others. A machine which fails is attended immediately by the service person unless she is busy fixing another machine. In that case, the broken machines form a queue. The time to repair a machine is $\text{Exp}(\lambda)$ distributed. Let Y_t be the number of operating machines at time t , so that $(Y_t)_{t \geq 0}$ is a pure jump process on $E = \{0, 1, \dots, M\}$. Determine the jump rate and the jump transition probability.

Exercise 7-3

M/M/1 queue with finite capacity. At a coiffeur only one customer can be served at a time, and there is also one chair on which the next customer can wait. Potential customers leave if they arrive at the shop when there are already two customers inside. The interarrival times and service times of the customers are independent and exponentially distributed with parameters λ and μ , respectively. Let X_t denote the number of customers in the shop at time t . Compute the jump rate and the jump transition probability.

There will be pre-exam office hours on July 18, 22, 25 and 29 at 13:00-14:30 in HG G 32.6.

Good luck!