6.1. Normalized Metrics. Let $\Phi(t) \in \text{Möb}^+(2)$ be a C^1-family such that $h(t) = e^{2v(t)} g_{\mathbb{S}^2}$ with

$$v(t) = u(t) \circ \Phi(t) + \tfrac{1}{2} \log(\det(d\Phi))$$

satisfies $\int_{\mathbb{S}^2} x \, d\mu_{h(t)} = 0$ as in Theorem 2.11.

Complete the proof of Theorem 2.11 by showing that there holds

$$\tfrac{dv}{dt}\big|_{t=t_0} = u_t \circ \Phi + dv \cdot \xi + \tfrac{1}{2} \operatorname{div}_{g_{\mathbb{S}^2}} \xi,$$

where $\xi = (d\Phi(t_0))^{-1} \, \tfrac{d}{dt}\big|_{t=t_0} \Phi$.