Probabilities and statistics

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Q1. We toss 100 times a coin and we get 60 head. We want to do a test to know whether the coin is fair.

(a) Test the hypothesis with a 0.01 level of significance. Should this test be one or two-tailed?.

(b) What is the biggest amount of head should we have in 100 tossings so we cannot discard $H_0 :=$ "The coin biased towards tail".

(c) Calculate all p_0 so that the null hypothesis

$$H_0(p_0) := \text{"Probability of head is } p_0 \text{ "},$$

would not be rejected in a test with 0.05 level of significance.
Hint: It will be useful to use the central limit theorem in all of this question.

Q2. Consider the null hypothesis $X \sim f(x)dx$ and the alternative $X \sim f(x-1)dx$ for the following cases:

$$f(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}},$$
$$f(x) = \frac{1}{\pi(1+x^2)}.$$

Compute the form of the rejection of the likelihood area ratio test (Neyman-Pearson Lemma). Comment the difference

Q3. Let $(X_i)_{i=1}^n$ be an i.i.d F-distributed sequence. Let F be absolutely continuous. The Sign test is a test where the null hypothesis is that the median of X is m, i.e.

$$F^{-1}(m) = \frac{1}{2}.$$

Use the Theorem 6.4 of the Skript to construct the test with significance level $\alpha = 0.05$.