

Probability Theory

Quiz 1

Question 1

State the Strong Law of Large Numbers.

Question 2

Compute the characteristic function ϕ_X of a random variable X with Binomial(n, p) ($p \in [0, 1]$ and $n \in \mathbb{Z}_{>0}$ fixed) distribution.

Question 3

Consider a sequence $\lambda_n > 0$ such that $\lambda_n \xrightarrow{n \rightarrow \infty} \infty$, and r.v. $(X_n)_{n \in \mathbb{N}}$, where X_n has exponential distribution with parameter λ_n : does the sequence $(X_n)_{n \in \mathbb{N}}$ converge in distribution? If so, what is the limit?

Question 4

Let $(X_i)_{i \in \mathbb{N}}$ be a sequence of i.i.d. random variables. What can you say about the σ -algebras $\sigma(X_i : i = 2n \text{ with } n \in \mathbb{N})$ and $\sigma(X_i : i = 2n + 1 \text{ with } n \in \mathbb{N})$?