Lösungen Prüfung

1 a) Löse $z^4 = -9 + 9\sqrt{3}\cdot i$ und zeichne die Lösungen

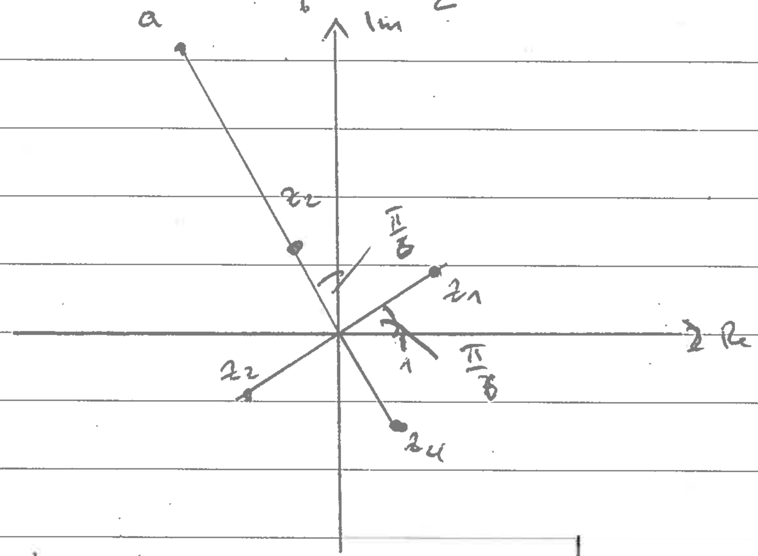
$$a = -9 + 9\sqrt{3}\cdot i = 18 \cdot e^{i\frac{2\pi}{3}}$$

$$z^4 = a \implies z_j = \sqrt[4]{18}\cdot e^{i\left(\frac{2\pi}{6} + j\cdot\frac{i\pi}{2}\right)} \qquad j = 0, \dots, 3$$

$$= \sqrt[4]{2}\cdot\sqrt{3}\cdot \exp\left[i\left(\frac{2\pi}{6} + j\frac{\pi}{2}\right)\right]$$

$$\cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2} \qquad \sin\left(\frac{\pi}{6}\right) = \frac{1}{2} \implies$$

$$\boxed{\begin{aligned} z_1 &= \tfrac{3}{2}\sqrt[4]{2} + i\cdot\tfrac{1}{2}\sqrt[4]{2}\cdot\sqrt{3} \\ z_2 &= -\tfrac{1}{2}\sqrt[4]{2}\sqrt{3} + \tfrac{3}{2}\sqrt[4]{2}\cdot i \\ z_3 &= -z_1 \\ z_4 &= -z_2 \end{aligned}}$$



b)

~~$\Pi = \{z \in \mathbb{C} : |z-(-2+2i)| \le |z-(3-i)|, \operatorname{Re} z < 0, \dots$~~

1/a) $z^4 = -9 + 9\sqrt{3}\,i = r \cdot e^{i\varphi}, \quad z = r_z e^{i\varphi_z}$

Winkel:

$\tan(\varphi) = \frac{9\sqrt{3}}{-9} = -\sqrt{3}$

$\overset{Re(z^4)<0}{\Longrightarrow} \quad \varphi = \frac{2}{3}\pi$

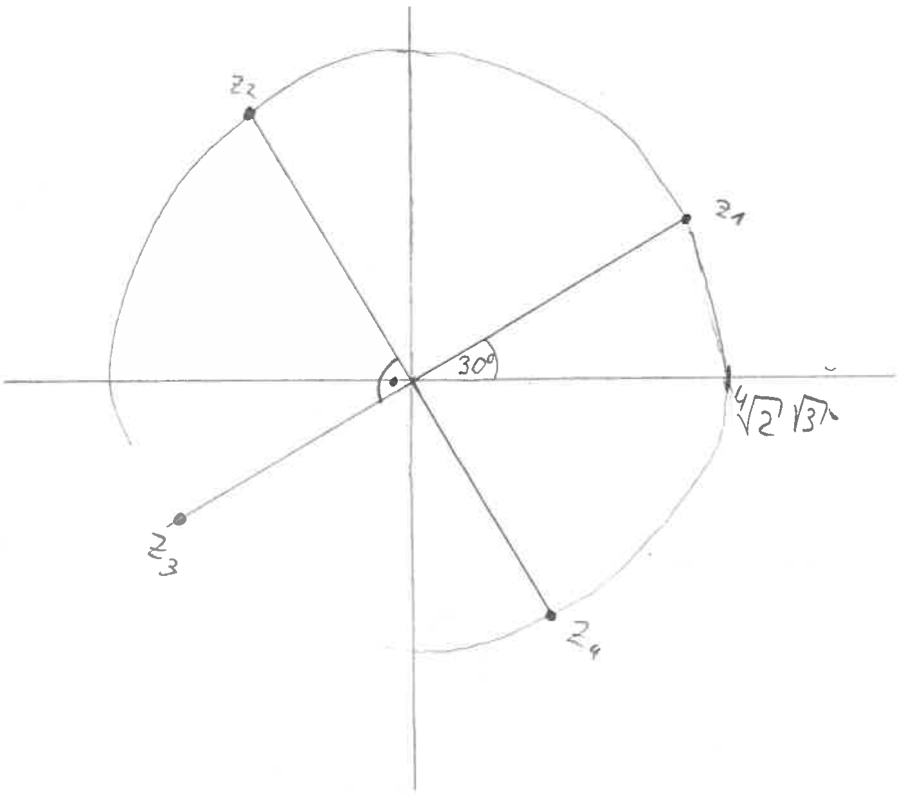
$\Rightarrow \varphi_z \in \left\{ \frac{1}{6}\pi, \frac{2}{3}\pi, -\frac{1}{3}\pi, -\frac{5}{6}\pi \right\}$

Betrag:

$r = \sqrt{(-9)^2 + (9\sqrt{3})^2} = 9 \cdot \sqrt{4} = 18$

$\Rightarrow r_z = \sqrt[4]{18} = \sqrt[4]{2}\sqrt{3}$

$$\boxed{z \in \left\{ \underbrace{\sqrt[4]{2}\sqrt{3}\,e^{\frac{1}{6}\pi i}}_{z_1}, \underbrace{\sqrt[4]{2}\sqrt{3}\,e^{\frac{2}{3}\pi i}}_{z_2}, \underbrace{\sqrt[4]{2}\sqrt{3}\,e^{-\frac{1}{3}\pi i}}_{z_4}, \underbrace{\sqrt[4]{2}\sqrt{3}\,e^{-\frac{5}{6}\pi i}}_{z_3} \right\}}$$



1/b) $M := \{ z \in \mathbb{C} \mid |z+1-2i| \leq |z-4+i| \wedge Re(z) < 0 \wedge Im(z) < 0 \}$

Alle komplexen Zahlen im 3. Quadranten, die zu $-1+2i$ näher sind als zu $4-i$, ohne Koordinatenachsen, aber inklusive Geradenabschnitt.