Afg. 8 (Viviani-Fenster)

Grundkreis Vollzylinder

$$D = \{(x,y) : (x-r)^2 + y^2 \le r^2\}$$
$$= \{(x,y) : x^2 + y^2 \le 2rx\}$$
$$= \{(\rho,\varphi) : \rho \le 2r\cos\varphi ,\ 0 \le \varphi \le \pi\}$$

Polarkoord.
$x = \rho\cos\varphi$
$y = \rho\sin\varphi$

Parametrisierung Viviani-Fenster: $\Phi(x,y) = \begin{bmatrix} x \\ y \\ \sqrt{4r^2 - x^2 - y^2} \end{bmatrix}$, $(x,y) \in$

(obere Halbkugel)

$$\Phi_x \times \Phi_y = \begin{bmatrix} 1 \\ 0 \\ -x/\sqrt{4r^2-x^2-y^2} \end{bmatrix} \times \begin{bmatrix} 0 \\ 1 \\ -y/\sqrt{4r^2-x^2-y^2} \end{bmatrix}$$

$$\|\Phi_x \times \Phi_y\|^2 = \frac{x^2}{4r^2-x^2-y^2} + \frac{y^2}{4r^2-x^2-y^2} + 1 = \frac{4r^2}{4r^2-x^2-y^2}$$

$$\text{Fläche} = \int_D \|\Phi_x \times \Phi_y\|\,dx\,dy = 2\int_D \|\Phi_x \times \Phi_y\|\,\rho\,d\rho\,d\varphi$$

$$= \int_0^{\pi}\int_0^{2r\cos\varphi} \frac{2r}{\sqrt{4r^2-\rho^2}}\,\rho\,d\rho\,d\varphi$$

$$= 2r\int_0^{\pi} \left[-\sqrt{4r^2-\rho^2}\right]_0^{2r\cos\varphi} d\varphi$$

$$= 2r\int_0^{\pi} (2r - 2r\sin\varphi)\,d\varphi$$

$$= 4r^2\pi - 16r^2 = \underline{\underline{4r^2(\pi-2)}}$$