Aufgabe 7

$$F = \{(x,y,z) : \; x+y+z=1, \; x,y,z \geq 0\}$$

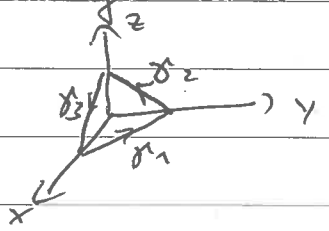
$$n = \frac{1}{\sqrt{3}}\begin{bmatrix}1\\1\\1\end{bmatrix} \qquad K(x,y,z) = \begin{bmatrix}x\\y\\z\end{bmatrix}$$

$$\operatorname{rot} K = \begin{bmatrix}0\\0\\0\end{bmatrix}$$

a) $$\int_F \underbrace{\operatorname{rot} K}_{=0} \cdot n \, do = 0$$

b) $$K = \frac{1}{2}\nabla(x^2+y^2+z^2)$$

$\Rightarrow$ Potentialfeld $\Rightarrow$ Integral über jede geschlossene Kurve 0.



Oder: Rand parametrisieren

$$\gamma_1(t) = \begin{bmatrix}1-t\\t\\0\end{bmatrix} \qquad 0 \leq t \leq 1 \qquad \dot{\gamma}_1 = \begin{bmatrix}-1\\1\\0\end{bmatrix}$$

$$\gamma_2(t) = \begin{bmatrix}0\\1-t\\t\end{bmatrix} \qquad 0 \leq t \leq 1 \qquad \dot{\gamma}_2 = \begin{bmatrix}0\\-1\\1\end{bmatrix}$$

$$\gamma_3(t) = \begin{bmatrix}t\\0\\1-t\end{bmatrix} \qquad 0 \leq t \leq 1 \qquad \dot{\gamma}_3 = \begin{bmatrix}1\\0\\-1\end{bmatrix}$$

$$\int_F \operatorname{rot} K \cdot n \, do = \int_{\partial F} K(\gamma(t)) \cdot \dot{\gamma}(t) \, dt$$

$$= 3 \cdot \int_0^1 (2t-1) \, dt = 0.$$