6) $f(x,y,z) = x+y-z$

$S = \{(x,y,z) \in \mathbb{R}^3 \mid x^2+3y^2=1, \; 4x=3z\}$

$g(x,y,z) := x^2+3y^2-1$

$h(x,y,z) := 4x-3z$

Lagrangemultiplikatoren:

$$\nabla f(x,y,z) + \lambda \nabla g(x,y,z) + \mu \nabla h(x,y,z) = 0$$

$$\begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix} + \lambda \begin{pmatrix} 2x \\ 6y \\ 0 \end{pmatrix} + \mu \begin{pmatrix} 4 \\ 0 \\ -3 \end{pmatrix} = 0$$

3. Zeile: $\mu = -\frac{1}{3}$

2. Zeile: $\lambda, y \neq 0, \quad \lambda = -\frac{1}{6y}$

1. Zeile: $1 - \frac{x}{3y} - \frac{4}{3} = 0$

$$x = -y$$

$$g(x,y,z) = 0 = (-y)^2 + 3y^2 - 1$$

$$y = \pm \frac{1}{2} \quad \Rightarrow \quad x = \mp \frac{1}{2}$$

$$\Rightarrow z = \mp \frac{2}{3}$$

$f(\frac{1}{2}, -\frac{1}{2}, -\frac{2}{3}) = \underline{\underline{\frac{2}{3}}} \rightarrow$ Max

$f(-\frac{1}{2}, \frac{1}{2}, \frac{2}{3}) = \underline{\underline{-\frac{2}{3}}} \rightarrow$ Min