3/a) $\int \frac{\ln(x)}{x}\,dx = \int \ln(x) \cdot \ln(x)'\,dx$

$= \int \frac{1}{2}\left(\ln(x)^2\right)'\,dx = \underline{\underline{\frac{1}{2}\ln(x)^2 + C}}$

b) $\int \frac{x+5}{x^3-2x^2+x}\,dx$

$x^3-2x^2+x = x(x^2-2x+1) = x(x-1)^2$

<u>Partialbruchzerlegung:</u>

$\frac{x+5}{x^3-2x^2+x} = \frac{A}{x} + \frac{Bx+C}{x^2-2x+1} = \frac{A(x^2-2x+1)+(Bx+C)}{x(x^2-2x+1)}$

$A + B = 0$

$-2A + C = 1$

$A = 5 \implies B = -5, \quad C = 11$

$\int \frac{5}{x}\,dx = 5\ln(x) + C$

$\int \frac{-5x+11}{x^2-2x+1}\,dx = -\frac{5}{2}\int \frac{2x-2}{x^2-2x+1}\,dx + \int \frac{6}{(x-1)^2}\,dx$

$= -\frac{5}{2}\ln(x^2-2x+1) - \frac{6}{x-1} + C$

$\int \frac{x+5}{x^3-2x^2+x}\,dx = \underline{\underline{5\ln(|x|) - \frac{5}{2}\ln(x^2-2x+1) - \frac{6}{x-1} + C}}$

c) $\int x(x+1)^{40}\,dx = x\,\frac{(x+1)^{41}}{41} - \int \frac{(x+1)^{41}}{41}\,dx$

$= x\,\frac{(x+1)^{41}}{41} - \frac{(x+1)^{42}}{41\cdot 42} + C$

$= \frac{(x+1)^{41}}{41}\left(x - \frac{x+1}{42}\right) + C$

$= \underline{\underline{\frac{(x+1)^{41}}{41}\left(\frac{41}{42}x - \frac{1}{42}\right) + C}}$

3 a) $\int \frac{\ln x}{x}\, dx = \int \ln x \,(\ln x)'\, dx = \underline{\frac{1}{2}\ln^2 x + C}$

b) $$\int \frac{x+5}{x^3-2x^2+x}\, dx = \int \left(\frac{A}{x} + \frac{B}{(x-1)^2} + \frac{C}{(x-1)}\right) dx$$

$A = \lim\limits_{x\to 0} \frac{x+5}{(x-1)^2} = 5$ $\qquad X + 5 - A(x-1)^2 - Bx = Cx(x-1)$

$\qquad\qquad\qquad\qquad\qquad = x+5-5x^2+4x-5$

$B = \lim\limits_{x\to 1} \frac{x+5}{x} = 6$ $\qquad = -5x(x-1)$

$C = -5$

$$\int \frac{x+5}{x^3-2x^2+x}\, dx = \int \frac{5}{x} + \frac{6}{(x-1)^2} - \frac{5}{(x-1)}\, dx$$

$$= 5\ln|x| - \frac{6}{x-1} - 5\ln|x-1| + C$$

$$= \underline{5\ln\left|\frac{x}{x-1}\right| - \frac{6}{x-1} + C}$$

c) $$\int x\cdot(x-1)^{41}\, dx = \frac{1}{41}\, x\,(x-1)^{41} - \frac{1}{41}\int (x-1)^{41}\, dx$$

$$= \frac{1}{41}\left[x - \frac{1}{42}(x-1)\right](x-1)^{41} + C$$

$$= \underline{\frac{1}{41\cdot 42}(41x-1)(x-1)^{41} + C}$$