Exercise 1

This exercise deals with line integrals and curve length, differentiability of complex-valued functions of one complex variable, analytic functions and the Cauchy-Riemann differential equations.

Due date: Tuesday, October 7^{th}, in the exercise class or at the latest at 3:00 pm in the boxes in room HG J 68. The online questions can be answered until Tuesday, October 7^{th}, at 3:00 pm.

1. Show that the length of a circle with radius R equals $2\pi R$ (via line integral).

2. Exercise 6, Page 46, from [Gam].

3. Exercise 8, Page 50, from [Gam].

4. Exercise 4, Page 106, from [Gam].

5. Exercise 10, Page 54, from [Gam].

6. Online questions

1. Let $D := \{z \in \mathbb{C} : \ |z| < 1\}$ be the open unit dist.. Which of the following expressions defines an analytic function $f : D \to \mathbb{C}$?

(a) $f(z) = \frac{z}{1+2z^2}$

(b) $f(z) = \frac{z}{1+2|z|^2}$

(c) $f(z) = \sum_{n=0}^{\infty} n! \ z^{n!}$

(d) $f(z) = \sum_{n=0}^{\infty} \sqrt{n}\, z^n$

(e) $f(z) = \frac{1}{\cos(z)}$

2. Let $f : \mathbb{C} \to \mathbb{C}$ be analytic. Which of the following $g : \mathbb{C} \to \mathbb{C}$ are also analytic?

(a) $g(z) = f(z)^2$

(b) $g(z) = f(z^2)$

(c) $g(z) = \overline{f(z)}$

(d) $g(z) = \overline{f(\bar{z})}$

(e) $g(z) = f(\bar{z})$

3. Let $u(X, Y) = X^2 + bXY + cY^2$, with $b, c \in \mathbb{R}$. Under which conditions is u the real part of an analytic function $f : \mathbb{C} \to \mathbb{C}$ (i.e. $f = u + iv$)?

(a) Always.

(b) Never.

(c) It depends on b.

(d) It depends on c.

4. Which pairs of functions (u, v) fulfill the Cauchy-Riemann equations?

(a) $u(x, y) = x + y, \quad v(x, y) = x + y$

(b) $u(x, y) = y, \quad v(x, y) = -x$

(c) $u(x, y) = \cos(x), \quad v(x, y) = \sin(x)$

(d) $u(x, y) = x^2 - y^2, \quad v(x, y) = 2xy$

(e) $u(x, y) = x^2 - y^2, \quad v(x, y) = -2xy$

(f) $u(x, y) = e^{U(x,y)} \cos(V(x, y)), v(x, y) = e^{U(x,y)} \sin(V(x, y))$, where U and V fulfill the Cauchy-Riemann equations.

(g) $u(x, y) = 3xy^2, \quad v(x, y) = y^3$