Exercise Sheet 8

This exercise sheet deals with the extended complex plane and the stereographic projection, as well as with residues in infinity and exterior domains.

Due date: Tuesday, December 2nd, in the exercise class or at the latest at 3:00 pm in the boxes in room HG J 68. The online questions can be answered until Tuesday, December 25nd, at 3:00 pm.

1. Exercise 4, Page 14, from [Gam].

2. Exercise 7, Page 15, from [Gam].

3. Exercise 4, Page 222, from [Gam].

4. Exercise 5, Page 222, from [Gam].

5. Use the generating function

$$G(t;\, z) = \frac{1}{\sqrt{1 - 2tz + z^2}} = \sum_{n=0} P_n(t)\, z^n$$

for Legendre polynomials and the binomial expansion

$$(1+x)^{-1/2} = \sum_{l=0}^{\infty} \frac{(-1)^l}{2^{2l}} \binom{2l}{l} x^l$$

to show that

$$P_n(t) = \frac{1}{2^n} \sum_{k=0}^{\lfloor n/2 \rfloor} (-1)^k \binom{n}{k} \binom{2n-2k}{n} t^{n-2k}.$$

6. Online questions

1. Let $\zeta = \exp(\frac{2\pi i}{5})$ and let γ be the five-pointed star consisting of the segments that connect $1, \zeta^2, \zeta^4, \zeta, \zeta^3, 1$ in this order Compute

$$\frac{1}{2\pi i}\int_\gamma \frac{z}{1-\cos(z)}dz.$$

(a) 0

(b) 1

(c) 2

(d) 3

(e) 4

(f) 5

(g) 6

2. Let $u : [0, 2\pi] \to \mathbb{R}$ with $u(t) = \frac{1}{\cos(t)-2}$. Which propositions are correct?

(a) $u(t) = f(e^{it})\, e^{it}$ for an analytic function $f : D \to \mathbb{C}$ on a domain D containing the unit circle.

(b) $\displaystyle\int_0^{2\pi} u(t)dt = 0.$

(c) $\displaystyle\int_0^{2\pi} u(t)dt = -2\pi/\sqrt{3}$

(d) $\displaystyle\int_0^{2\pi} u(t)dt = -2\pi$