Exercise Sheet 4

Exercise 1

Let G be a connected topological group. Show that every neighborhood of the identity element e generates G as an abstract group.

Exercise 2

(a) Show that the Lie groups $\mathcal{S}^3 = \mathrm{Sp}(1)$ and $\mathrm{SU}(2)$ are isomorphic.

(b) Show that the Lie groups $\mathcal{S}^3/\{1,-1\}$ and $\mathrm{SO}(3)$ are isomorphic.

Exercise 3

Show that for two left-invariant vector fields X, Y on $\mathrm{GL}(n,\mathbb{R})$, $[X,Y]_e = X_eY_e - X_eY_e$ (matrix product).

Exercise 4

Show that the Lie algebra $\mathbb{R}^3$ with the bracket given by the vector cross product is isomorphic to the Lie algebra of $\mathrm{O}(3,\mathbb{R})$.