Exercise Sheet 5

Exercise 1

Show that the exponential map of $\mathrm{GL}(n, \mathbb{C})$ is surjective.

Exercise 2

Show that the exponential map of the identity component $\mathrm{GL}(n, \mathbb{R})_0$ of $\mathrm{GL}(n, \mathbb{R})$ is not surjective.

Exercise 3

Let G be a Lie group. Use properties of the exponential map to show that there is a neighborhood of the identity element e_G in G that contains no non-trivial subgroup of G.

Exercise 4

Check, by direct computation, that the exponential map of the three-dimensional Heisenberg group N of unipotent upper-triangular matrices satisfies

$$\exp(A)\exp(B) = \exp(A + B + \frac{1}{2}[A, B])$$

for all A, B in TN_e.