Exercisesheet 6

1. a) Calculate the inverse arcosh of the hyperbolic cosine function

$$\cosh\colon [0,\infty] \to [1,\infty)\colon x \mapsto \frac{e^x + e^{-x}}{2}.$$

b) Determine the derivative of arcosh first directly and then using the chain rule.

2. Newton's Method:

a) Using Newton's Method we want to find a numerical approximation for $\sqrt{2}$, which is a root of $f(x) = x^2 - 2$. Find the recursion formula for x_n and calculate the first three values with starting point $x_0 = 1$.

b) Set up Newton's Method for $f(x) = x^3 - 2x + 2$ and starting point $x_0 = 0$. Calculate the first three iteration steps. How can you explain this behaviour?

c) For Newton's Method we assume that f is differentiable and that we know its derivative f'. Similarly to Newton's Method we now deduce the Secant Method which only uses the values of f. Given two points x_n, x_{n-1} we obtain our new value x_{n+1} by intersecting the secant line going through $(x_n, f(x_n))$ and $(x_{n-1}, f(x_{n-1}))$ with the x-axes.

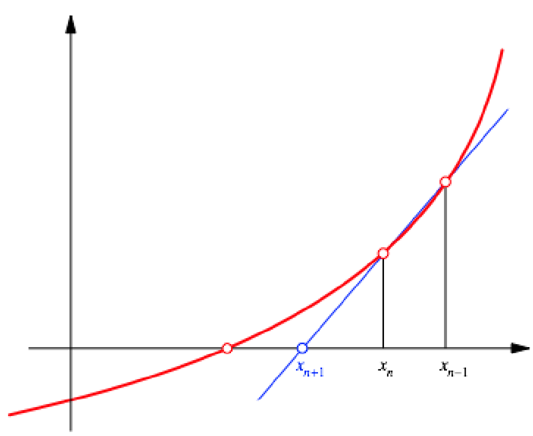


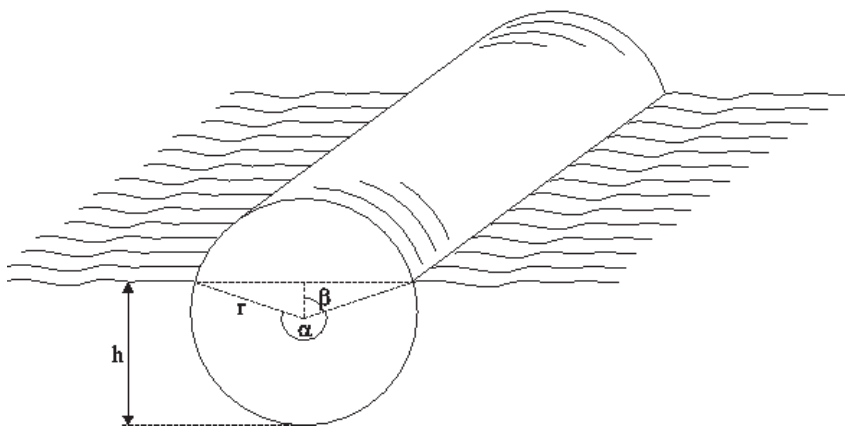
Abbildung 1: Sekant Method

Bitte wenden!

i. Find an equation for the secant line.
ii. Deduce the recursion formula for x_{n+1}.
iii. Calculate the first three approximation values of $\sqrt{2}$ using the starting points $x_0 = 0, x_1 = 1$.

3. Concider a wooden cylinder of length l and radius r swimming in water (density $\varrho_0 = 1\, g/cm^3$). The wood has density ϱ_1.

Using Archimedes' principle (the weight of a body immersed in a fluid is equal to the weight of the fluid it displaces) we can establish a formula for deducing the depth of immersion h or rather the opening angle α (see image):



$$r^2\pi \cdot l\varrho_1 = \left[\frac{r^2}{2}\alpha + r^2 \sin\underbrace{\left(\pi - \frac{\alpha}{2}\right)}_{=\beta} \cdot \cos\underbrace{\left(\pi - \frac{\alpha}{2}\right)}_{=\beta}\right] \cdot l\varrho_0 \tag{1}$$

Multiplication with $\frac{2}{r^2\varrho_0 l}$ and the definition of the density quotient $\varrho = \frac{\varrho_1}{\varrho_0} \in (0,1)$ leads to

$$\alpha - \sin\alpha = 2\pi\varrho. \tag{2}$$

This equality has no analytic solution, but we can solve it numerically. You may use a calculator.

a) Prove equation (1).

b) Deduce equation (2) from equation (1) as described.

c) Reformulate equation (2) such that it is of the form $f(\alpha) = 0$.

d) Draw the function $f(\alpha)$ for $\varrho = 0.8$ (oak). Also add the tangent line at the point $(\alpha_0, f(\alpha_0))$, where $\alpha_0 = 5$.

Siehe nächstes Blatt!

e) What is the formula for the tangent line? Determine the intersection point α_1 of the tangent line and the x-axes. (This is the first step in Newton's Method.)

f) Proceed by calculating five more iteration steps of Newton's Method.

g) How big is the depth of immersion approximately?

4. Repetition-Exercise. Calculate the derivatives of the following functions:

a) $f(x) = x \sin x$	**b)** $f(x) = \dfrac{1}{x^2+3}$	**c)** $f(x) = \dfrac{1}{\sin x}$
d) $f(x) = e^{-x}$	**e)** $f(x) = (\tan x)^2$	**f)** $f(x) = \sqrt{\ln x}$
g) $f(x) = \tan\left(\dfrac{1+x}{1-x}\right)$	**h)** $f(x) = \sqrt{x^2 + \sqrt{x^2+1}}$	