Exercisesheet 12

1. Consider the following parametrisation of the hyperbola $x^2 - y^2 = 1$ (see figure)

$$r : \mathbb{R} \to \mathbb{R}^2, \quad r(t) = \begin{pmatrix} x(t) \\ y(t) \end{pmatrix} = \begin{pmatrix} \cosh t \\ \sinh t \end{pmatrix}.$$

a) Calculate the grey marked area in figure 1 which is bounded by the x-axis, the hyperbola and the straight line from the origin to the point $r(T)$.

Hint: Use the formula

$$F = \frac{1}{2} \int_{t_1}^{t_2} \big(x(t)\dot{y}(t) - \dot{x}(t)y(t) \big) \, \mathrm{d}t.$$

Choose the bounds of integration correctly.

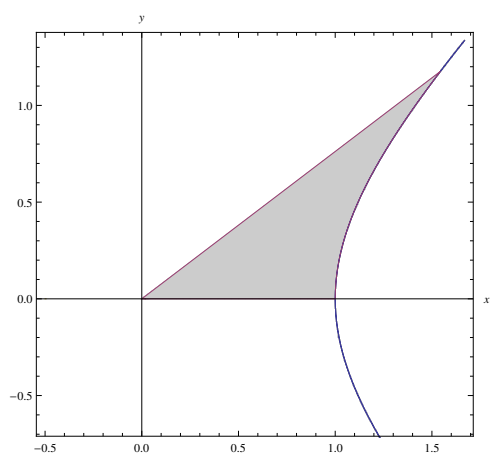

Abbildung 1: The hyperbola from Exercise 1. The sought-after area is marked grey.

b) What is the relation between the area and the parameter T?

2. **a)** Calculate the length of the curve given by

$$r(t) = \begin{pmatrix} x(t) \\ y(t) \end{pmatrix} = \begin{pmatrix} \mathrm{e}^{-t}\cos(5t) \\ \mathrm{e}^{-t}\sin(5t) \end{pmatrix}, \quad t \in [0, 2\pi].$$

The curve is drawn on the left hand side of figure 2.

Bitte wenden!

b) Determine the length of the curve which satisfies the following equation:

$$y^2 = 1 - |x|$$

The curve is shown on the right hand side of figure 2.

c) What is the length of the curve in part a) if we repace $t \in [0, 2\pi]$ by $t \in [0, \infty)$?

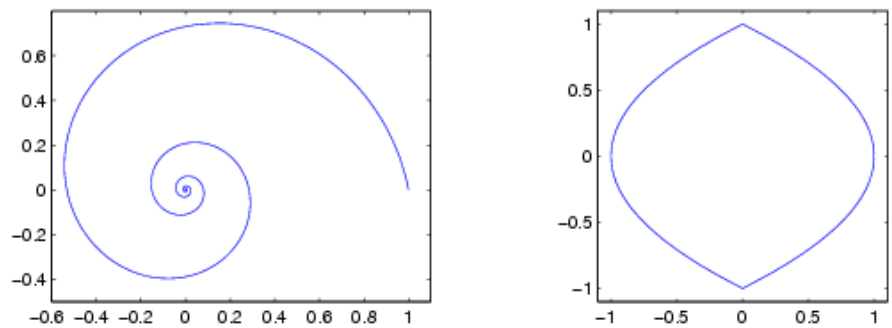

Abbildung 2: The curves of exercises 2a) (left) und 2b) (right).

3. A spinner is generated by rotating the function

$$f(x) = x^{\frac{3}{5}}, \qquad 0 \leq x \leq 1$$

around the y-axis. The mass density inside the spinner is

$$\varrho(y) = 2 - y, \qquad 0 \leq y \leq 1.$$

a) Compute the mass of the spinner.

b) What is the height of the center of mass?

4. Calculate the surface and the volume of a ball of radius 1 which has a cylindrical hole of radius $a < 1$ going through the center.