Musterlösung Lineare Algebra und Numerische Mathematik

Winter 2010 Prof. D. Stoffer

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1. a)

$$\left(\begin{array}{ccc|c} 2 & 3 & -5 & -1 \\ 6 & 10 & -9 & 5 \\ -4 & 0 & 4 & 3 \end{array}\right) \sim \left(\begin{array}{ccc|c} 2 & 3 & -5 & -1 \\ 0 & 1 & 6 & 8 \\ 0 & 6 & -6 & 1 \end{array}\right)$$

b) Pivots: 2, $a-2$, a

• $a \in \mathbb{R} \setminus \{0,2\}$:

$$\left(\begin{array}{ccc|c} 2 & 2-a & -2a & b \\ 0 & a-2 & a & a-1 \\ 0 & 0 & a & b \end{array}\right)$$

$$\Downarrow$$

$$x_3 = b/a$$

$$\Rightarrow x_2 = \frac{1}{a-2} \cdot (a-1 - a \cdot b/a) = \frac{1}{a-2} \cdot (a-1-b)$$

$$= \frac{a-b-1}{a-2}$$

$$\Rightarrow x_3 = \frac{1}{2}\left(b - (-2a) \cdot b/a - (2-a) \cdot \frac{a-b-1}{a-2}\right)$$

$$= \frac{1}{2} \cdot (\cancel{b} + 2b + a - \cancel{b} - 1) = \frac{1}{2}(a+2b-1)$$

$$\Rightarrow \mathbb{L} = \left\{ \left(b/a, \frac{a-b-1}{a-2}, \frac{a+2b-1}{2} \right)^T \right.$$

• $a = 0$:

$$\left(\begin{array}{ccc|c} 2 & 2 & 0 & b \\ 0 & -2 & 0 & -1 \\ 0 & 0 & 0 & b \end{array}\right) \Rightarrow \text{Verträglichkeits-Bedingung } b = 0.$$

$$\Rightarrow x_3 = t$$

$$\Rightarrow x_2 = \frac{1}{2}$$

$$\Rightarrow x_1 = \frac{1}{2} \cdot (\underset{0}{\overset{\shortparallel}{b}} - 2 \cdot \tfrac{1}{2}) = \frac{1}{2} \cdot -1 = -\frac{1}{2}$$

∘ $b = 0$: $\mathbb{L} = \left\{ \begin{pmatrix} -1/2 \\ 1/2 \\ t \end{pmatrix} \mid t \in \mathbb{R} \right\}$

∘ $b \neq 0$: $\mathbb{L} = \emptyset$

• $a=2$:

$$\left(\begin{array}{ccc|c} 2 & 0 & -4 & b \\ 0 & 0 & 2 & 1 \\ 0 & 0 & 2 & b \end{array}\right) \sim \left(\begin{array}{ccc|c} 2 & 0 & -4 & b \\ 0 & 0 & 2 & 1 \\ 0 & 0 & 0 & b-1 \end{array}\right) \Rightarrow VB \; \underline{b=1}$$

$\Rightarrow x_3 = 1/2$

$\Rightarrow x_2 = t$

$\Rightarrow x_1 = \frac{1}{2}(\underset{=1}{b} + 4 \cdot 1/2) = \frac{1}{2}(3) = 3/2$

• $b=1$: $\mathbb{L} = \{(3/2, t, 1/2)^T \mid t \in \mathbb{R}\}$

• $b \neq 1$: $\mathbb{L} = \emptyset$

$\boxed{2.}$ $\det(A) = \begin{vmatrix} -9 & 6 & -6 \\ 6 & 4 & 0 \\ -6 & 0 & -2 \end{vmatrix} = (-9)\cdot 4(-2) + 0 + 0 - (-6)\,4(-6) - 0 - 6\cdot 6\cdot(-2)$

$= 9\cdot 8 - 4\cdot 36 + 2\cdot 36 = 9\cdot 8 - 2\cdot 36$

$= 72 - 72 = 0$

a) Falsch
b) Wahr
c) Falsch
d) Wahr
e) Wahr
f) $\begin{pmatrix} -9 & 6 & -6 \\ 6 & 4 & 0 \\ -6 & 0 & -2 \end{pmatrix} \cdot \begin{pmatrix} 3 \\ 6 \\ -2 \end{pmatrix} = \begin{pmatrix} -27+36+12 \\ 18+24 \\ -18+4 \end{pmatrix} = \begin{pmatrix} 48-27 (=21) \\ 42 \\ -14 \end{pmatrix}$

$= 7 \cdot \begin{pmatrix} 3 \\ 6 \\ -2 \end{pmatrix}$

$\Rightarrow$ f) Wahr
g) Wahr
h) Wahr

$\boxed{3.}$ a) $B_1 = \begin{pmatrix} 0.5 \\ 0.5 \\ 0 \end{pmatrix}, \quad B_2 = \begin{pmatrix} 0 \\ 0.5 \\ 0.5 \end{pmatrix}, \quad B_3 = \begin{pmatrix} 0.5 \\ 0 \\ 0.5 \end{pmatrix}$

$$\Rightarrow A = \begin{pmatrix} 1/2 & 0 & 1/2 \\ 1/2 & 1/2 & 0 \\ 0 & 1/2 & 1/2 \end{pmatrix}, \quad \text{denn} \quad A \cdot E_j = B_j$$

b) Geometrisch: A ist eine Rotation um den Vektor $\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$

$\Rightarrow$ alle Punkte $\{ t \cdot \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \mid t \in \mathbb{R} \}$ sind Fixpunkte.

Rechnerisch:

$$A \cdot x = x \Leftrightarrow (A - I) \cdot x = 0$$

$$\left(\begin{array}{ccc|c} -1/2 & 0 & 1/2 & 0 \\ 1/2 & -1/2 & 0 & 0 \\ 0 & 1/2 & -1/2 & 0 \end{array}\right) \rightsquigarrow \left(\begin{array}{ccc|c} -1/2 & 0 & 1/2 & 0 \\ 0 & -1/2 & 1/2 & 0 \\ 0 & 1/2 & -1/2 & 0 \end{array}\right)$$

$$\rightsquigarrow \left(\begin{array}{ccc|c} -1/2 & 0 & 1/2 & 0 \\ 0 & -1/2 & 1/2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

$x_3 = t$

$\Rightarrow x_2 = t \qquad \Rightarrow \mathbb{L} = \{ \begin{pmatrix} t \\ t \\ t \end{pmatrix} \mid t \in \mathbb{R} \}$

$\Rightarrow x_1 = t$

c) $$n_{\alpha_0} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \cdot \frac{1}{\left\| \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \right\|} = \frac{1}{\sqrt{3}} \cdot \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

$$\Rightarrow P = I - n_{\alpha_0} n_{\alpha_0}^T = I - \frac{1}{3} \cdot \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} = \begin{pmatrix} 2/3 & -1/3 & -1/3 \\ -1/3 & 2/3 & -1/3 \\ -1/3 & -1/3 & 2/3 \end{pmatrix}$$

alternativ: Finde den Punkt $t \cdot \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$, der auf α liegt:

$$\begin{pmatrix} t \\ t \\ t \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + \mu \cdot \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix} + \nu \cdot \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 - \mu - \nu \\ \mu \\ \nu \end{pmatrix}$$

$\Rightarrow \mu = \nu = 1 - \mu - \nu \Rightarrow \mu = 1 - 2\mu \Rightarrow 3\mu = 1 \Rightarrow \mu = \nu = 1/3$

$\Rightarrow$ im Punkt $\begin{pmatrix} 1/3 \\ 1/3 \end{pmatrix}$

Die Punkte $E_j' = E_j - \begin{pmatrix} 1/3 \\ 1/3 \\ 1/3 \end{pmatrix}$ sind die Bildpunkte von E_1, E_2, E_3 durch P:

$$E_1' = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} - \begin{pmatrix} 1/3 \\ 1/3 \\ 1/3 \end{pmatrix} = \begin{pmatrix} 2/3 \\ -1/3 \\ -1/3 \end{pmatrix}, \quad E_2' = \begin{pmatrix} -1/3 \\ 2/3 \\ -1/3 \end{pmatrix}, \quad E_3' = \begin{pmatrix} -1/3 \\ -1/3 \\ 2/3 \end{pmatrix}.$$

Man findet P wie in (a).

d) geometrisch: $z_{k+1} = A \cdot z_k$ bewirkt eine Drehung um $\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$, und eine Verkleinerung des Abstands zur Geraden $\left\{ \begin{pmatrix} t \\ t \\ t \end{pmatrix} \middle| t \in \right.$
Also ist der Fixpunkt $\begin{pmatrix} 1/3 \\ 1/3 \\ 1/3 \end{pmatrix}$.

rechnerisch: Eigenwerte von A bestimmen:

$$\det \begin{pmatrix} 1/2-\lambda & 0 & 1/2 \\ 1/2 & 1/2-\lambda & 0 \\ 0 & 1/2 & 1/2-\lambda \end{pmatrix} = \left(\tfrac{1}{2}-\lambda\right)^3 + \tfrac{1}{8} - 0 - 0 - 0$$

$$= \cancel{\tfrac{1}{8}} \left(\tfrac{1}{4} - \lambda + \lambda^2\right)\left(\tfrac{1}{2} - \lambda\right) + \tfrac{1}{8}$$

$$= \tfrac{1}{8} - \tfrac{1}{2}\lambda + \tfrac{1}{2}\lambda^2 - \tfrac{1}{4}\lambda + \lambda^2 - \lambda^3 + \tfrac{1}{8}$$

$$= \tfrac{1}{4} - \tfrac{3}{4}\lambda + \tfrac{3}{2}\lambda^2 - \lambda^3$$

$\lambda = 1$ Eigenwert, und $-(\lambda-1)\cdot(\lambda-a)(\lambda-b) = (\lambda^2 - (1+a)\lambda + a)(\lambda - b)$

$$= -\lambda^3 + (1+a)\lambda^2 - a\lambda + b\lambda^2 - b(1+a)\lambda + ab$$

$$= -\lambda^3 + (a+b+1)\lambda^2 - (a+b+ab)\lambda + ab$$

$\Rightarrow$ $ab = \frac{1}{4}$, $a+b = \frac{3}{2} - 1 = \frac{1}{2}$

$\Rightarrow a \cdot \left(\frac{1}{2} - a\right) = \frac{1}{4}$

$\Leftrightarrow \frac{1}{2}a - a^2 - \frac{1}{4} = 0 \Leftrightarrow -4a^2 + 2a - 1 = 0$

$\Leftrightarrow a_{1,2} = \dfrac{-2 \pm \sqrt{4 - 4(-4)(-1)}}{2\cdot(-4)}$

$= \dfrac{-2 \pm \sqrt{4-16}}{-8} = \dfrac{1 \pm \sqrt{-3}}{4}$

$\Rightarrow \lambda_1 = 1, \ \lambda_2 = \dfrac{1+\sqrt{3}\,i}{4}, \ \lambda_3 = \dfrac{1-\sqrt{3}\,i}{4}$

$|\lambda_2| = |\lambda_3| = \frac{1}{4} \cdot \sqrt{1+3} = \frac{2}{4} = \frac{1}{2} < 1.$

$$\Rightarrow \quad A = U D U^{-1}, \quad D = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{pmatrix}, \quad U = (v_1, v_2, v_3)$$

$$z_k = A^k \cdot \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = U \cdot D^k \cdot \underbrace{U^{-1} \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}}_{=y}$$

$$\lim_{k\to\infty} D^k = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \Rightarrow \lim_{k\to\infty} z_k = U \cdot \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \cdot y = v_1 \cdot \underbrace{y_1}_{\text{Skalar}}$$

$\Rightarrow$ finde v_1:

$$\left(\begin{array}{ccc|c} -1/2 & 0 & 1/2 & 0 \\ 1/2 & -1/2 & 0 & 0 \\ 0 & 1/2 & -1/2 & 0 \end{array}\right) \sim \begin{pmatrix} -1/2 & 0 & 1/2 \\ 0 & -1/2 & 1/2 \\ 0 & 1/2 & -1/2 \end{pmatrix}$$

aus b) $v_1 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \Rightarrow \lim_{k\to\infty} z_k = \begin{pmatrix} t \\ t \\ t \end{pmatrix}$

Da dieser Vektor auf α liegen muss, gilt

$$\begin{pmatrix} t \\ t \\ t \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + \mu \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix} + \nu \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1-\mu-\nu \\ \mu \\ \nu \end{pmatrix}$$

$$\Rightarrow \mu = \nu, \quad \mu = 1 - 2\mu \Rightarrow \mu = \frac{1}{3} \Rightarrow t = \frac{1}{3} \Rightarrow \lim_{k\to\infty} z_k = \begin{pmatrix} 1/3 \\ 1/3 \\ 1/3 \end{pmatrix}$$

$\rightarrow$ Mühsamer Weg!

$\boxed{4.}$ $K(x) = a \cdot \left(\frac{x^2}{2} - \frac{x}{2} + 1 \right) + b \cdot \left(x^2 - 2x - 1 \right)$

$$\Rightarrow A = \begin{pmatrix} \frac{1}{2} - \frac{-1}{2} + 1 & 1 + 2 - 1 \\ 1 & -1 \\ \frac{1}{2} - \frac{1}{2} + 1 & 1 - 2 - 1 \\ \frac{4}{2} - 1 + 1 & 4 - 4 - 1 \end{pmatrix} = \begin{pmatrix} 2 & 2 \\ 1 & -1 \\ 1 & -2 \\ 2 & -1 \end{pmatrix}, \quad b = \begin{pmatrix} 5 \\ 1 \\ 0 \\ 2 \end{pmatrix}$$

$$A^T A = \begin{pmatrix} 4+1+1+4 & 4-1-2-2 \\ -1 & 4+1+4+1 \end{pmatrix} = \begin{pmatrix} 10 & -1 \\ -1 & 10 \end{pmatrix}$$

$$A^T b = \begin{pmatrix} 10+1+4 \\ 10-1-2 \end{pmatrix} = \begin{pmatrix} 15 \\ 7 \end{pmatrix}$$

$$\left(\begin{array}{cc|c} 10 & -1 & 15 \\ -1 & 10 & 7 \end{array}\right) \rightsquigarrow \left(\begin{array}{cc|c} 1 & -10 & -7 \\ 10 & -1 & 15 \end{array}\right) \rightsquigarrow \left(\begin{array}{cc|c} 1 & -10 & -7 \\ 0 & 99 & 85 \end{array}\right)$$

$$\Rightarrow \underline{\underline{b = \frac{85}{99}}}, \quad \underline{a} = -7 + 10 \cdot \frac{85}{99} = \frac{-7 \cdot 99 + 850}{99}$$

$$= \frac{850 - 693}{99} = \underline{\frac{157}{99}}$$

$\boxed{5.}$ a) $\dot{x} = 0 \Rightarrow Ax = -b$

$$\begin{array}{cc|c} 1 & 2\sqrt{3} & 0 \\ 2\sqrt{3} & 5 & -14 \end{array} \sim \begin{array}{cc|c} 1 & 2\sqrt{3} & 0 \\ 0 & -7 & -14 \end{array}$$

$\Rightarrow x_2 = 2$

$\Rightarrow x_1 = -2\sqrt{3} \cdot 2 = -4\sqrt{3}$

$$\Rightarrow x^{GG}(t) = \begin{pmatrix} -4\sqrt{3} \\ 2 \end{pmatrix}$$

b) $$\left| \begin{pmatrix} 1-\lambda & 2\sqrt{3} \\ 2\sqrt{3} & 5-\lambda \end{pmatrix} \right| = (1-\lambda)(5-\lambda) - 12 = 5 - 6\lambda + \lambda^2 - 12$$

$$= \lambda^2 - 6\lambda - 7 \overset{!}{=} 0$$

$$\lambda_{1/2} = \frac{6 \pm \sqrt{36 - 4 \cdot (-7)}}{2} = 3 \pm \sqrt{9+7} = 3 \pm 4 = 7 / -1$$

$(\lambda+1)\cdot(\lambda-7) = \lambda^2 - 6\lambda - 7$ ✓

$$v_1: \begin{array}{cc|c} 2 & 2\sqrt{3} & 0 \\ 2\sqrt{3} & 6 & 0 \end{array} \sim \begin{array}{cc|c} 2 & 2\sqrt{3} & 0 \\ 0 & 0 & 0 \end{array} \Rightarrow v_1 = \begin{pmatrix} \sqrt{3} \\ -1 \end{pmatrix}$$

$$v_7: \begin{array}{cc|c} -6 & 2\sqrt{3} & 0 \\ 2\sqrt{3} & -2 & 0 \end{array} \sim \begin{array}{cc|c} 0 & 0 & 0 \\ 2\sqrt{3} & -2 & 0 \end{array} \Rightarrow v_7 = \begin{pmatrix} 1 \\ \sqrt{3} \end{pmatrix}$$

$$\Rightarrow U = \begin{pmatrix} \sqrt{3} & 1 \\ -1 & \sqrt{3} \end{pmatrix}, \quad D = \begin{pmatrix} -1 & 0 \\ 0 & 7 \end{pmatrix}$$

c) $x^{hom}(t) = ?$ $\qquad \dot{x}^{hom}(t) = A \cdot x^{hom}(t) = U D U^{-1} x^{hom}(t)$

$$\Rightarrow U^{-1} \dot{x}^{hom}(t) = D U^{-1} x^{hom}(t) = D y(t)$$

$$\Rightarrow y(t) = \begin{pmatrix} y_1 \cdot e^{-t} \\ y_2 \cdot e^{7t} \end{pmatrix}, \quad x^{hom}(t) = U \cdot y(t) = \begin{pmatrix} \sqrt{3} y_1 e^{-t} + y_2 e^{7t} \\ -y_1 e^{-t} + \sqrt{3} y_2 e^{7t} \end{pmatrix}$$

$$\Rightarrow x(t) = x^{GG}(t) + x^{hom}(t) = \begin{pmatrix} -4\sqrt{3} \\ 2 \end{pmatrix} + \begin{pmatrix} \sqrt{3}\, y_1 e^{-t} + y_2 e^{7t} \\ -y_1 e^{-t} + \sqrt{3}\, y_2 e^{7t} \end{pmatrix}$$

$$\text{AB: } x(0) = \begin{pmatrix} -4\sqrt{3} + \sqrt{3}\, y_1 + y_2 \\ 2 - y_1 + \sqrt{3}\, y_2 \end{pmatrix} \overset{!}{=} \begin{pmatrix} -3\sqrt{3} \\ 9 \end{pmatrix}$$

$$\text{d.h. } \begin{cases} \sqrt{3}\, y_1 + y_2 = \sqrt{3} \\ -y_1 + \sqrt{3}\, y_2 = 7 \end{cases}$$

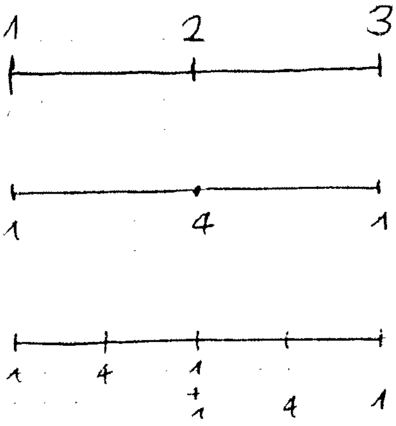
$$\left(\begin{array}{cc|c} \sqrt{3} & 1 & \sqrt{3} \\ -1 & \sqrt{3} & 7 \end{array}\right) \rightsquigarrow \left(\begin{array}{cc|c} 1 & -\sqrt{3} & -7 \\ 0 & 4 & \sqrt{3} + 7\sqrt{3} = 8\sqrt{3} \end{array}\right)$$

$$\Rightarrow \cancel{x}\, y_2 = 2\sqrt{3}$$

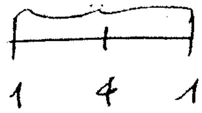
$$\Rightarrow y_1 = -7 + \sqrt{3} \cdot 2\sqrt{3} = -7 + 6 = -1$$

$$\Rightarrow x(t) = \begin{pmatrix} -4\sqrt{3} \cancel{+} -\sqrt{3}\, e^{-t} + 2\sqrt{3}\, e^{7t} \\ 2 + e^{-t} + 6 e^{7t} \end{pmatrix} = \begin{pmatrix} \sqrt{3} \cdot (-4 - e^{-t} + 2e^{7t}) \\ 2 + e^{-t} + 6 e^{7t} \end{pmatrix}$$

6. a)



Simpson:



$$\hat{I}(2) = \frac{2}{6}\cdot\left(1\cdot\frac{1}{1} + 4\cdot\frac{1}{2} + 1\cdot\frac{1}{3}\right) = \frac{2}{6}\cdot\left(1+2+\frac{1}{3}\right) = \frac{2}{6}\cdot\frac{9+1}{3} = \frac{20}{18} = \underline{\underline{\frac{10}{9}}}$$

$$\approx 1.11\overline{1}$$

$$\hat{I}(1) = \frac{2}{12}\cdot\left(1\cdot\frac{1}{1} + 4\cdot\frac{1}{1.5} + 2\cdot\frac{1}{2} + 4\cdot\frac{1}{2.5} + 1\cdot\frac{1}{3}\right)$$

$$= \frac{1}{6}\cdot\left(1 + 4\frac{2}{3} + 1 + 4\frac{2}{5} + \frac{1}{3}\right) = \frac{1}{6}\cdot\left(2 + \underbrace{\frac{1}{3} + \frac{8}{3}}_{=3} + \frac{8}{5}\right) = \frac{1}{6}\left(\frac{25+8}{5}\right)$$

$$= \frac{33}{30} = \underline{\underline{\frac{11}{10}}}$$

$$\approx$$

b) $\hat{I}(1) = I + c_4\cdot 1^4 + c_5\cdot 1^5 + O(h^6)$

$\hat{I}(2) = I + c_4\cdot 2^4 + c_5\cdot 2^5 + O(h^6)$

$$\Rightarrow 2^4\cdot\hat{I}(1) - \hat{I}(2) = 2^4 I + 2^4 c_4 + 2^4 c_5 + 2^4 O(h^6)$$

$$- I - 2^4 c_4 - 2^5 c_5 + O(h^6)$$

$$= (16-1)\, I + 0\cdot c_4 + (16-32)\, c_5 + O(h^6)$$

$$\Rightarrow I = \frac{1}{15}\cdot\left[\left(2^4\hat{I}(1) - \hat{I}(2)\right) + 16\, c_5 + O(h^6)\right]$$

$$\approx I^{ext} = \frac{1}{15}\cdot\left(16\,\hat{I}(1) - \hat{I}(2)\right) = \frac{1}{15}\left(16\cdot\frac{11}{10} - \frac{10}{9}\right)$$

$$= \frac{1}{15}\left(\frac{16\cdot 99}{90} - \frac{100}{90}\right) = \frac{1}{15}\cdot\frac{990 + 594 - 100}{90}$$

$$= \frac{1}{15}\,\frac{\cancel{1484}^{742}}{\cancel{90}_{45}} = \frac{7\cdot 106}{3\cdot 5\cdot 5\cdot 9} = \frac{7\cdot 2\cdot 53}{5\cdot 5\cdot 3\cdot 3\cdot 3} = \underline{\underline{\frac{742}{675}}}$$

$$\approx$$

c) $\left|\frac{\hat{I}(1) - \ln(3)}{\ln(3)}\right| \approx 1{,}1377\cdot 10^{-2}$

$\left|\frac{I^{ext} - \ln(3)}{\ln(3)}\right| \approx 5{,}88898\cdot 10^{-4}$

$\left|\frac{\hat{I}(2) - \ln(3)}{\ln(3)}\right| \approx 1.2631\cdot 10^{-3}$