

## Problem set – Week 4

## MORE INTEGRATION PROBLEMS, PATH INTEGRALS

1. Compute the following integrals.

$$(a) \int x \log x \, dx \quad (b) \int \frac{dx}{x^2 \sqrt{x^2 + 1}} \quad (c) \int \frac{dx}{x^2(x^2 - 1)}$$

2. Sketch the region enclosed by the line  $x = 4$ , the curve  $y = \sqrt{x}$  and the  $x$ -axis. Compute its area. Do the same for the region in the first quadrant that is bounded by  $y = x^3$  and  $y = 4x$ .

3. Compute the following integrals.

$$(a) \int_0^\infty \frac{e^{-\sqrt{x}}}{\sqrt{x}} \quad (b) \int_e^\infty \frac{dx}{x \log x} \quad (c) \int_0^3 \frac{x \, dx}{(x^2 - 1)^{2/3}}$$

4. For which  $x \in (0, 3\pi/2)$  is  $f(x) = \int_x^{2x} \frac{\sin t}{t} dt$  a local maximum ?

5. Compute the length of the curve defined by  $y = \sqrt{x^3}$  on the interval  $0 \leq x \leq 28$ .

6. Compute the line integral of  $x + y^2$  over the segment of the circle  $x^2 + y^2 = 4$  going from  $(2, 0)$  to  $(0, 2)$ . Then compute again this line integral but going this time from  $(0, 2)$  to  $(2, 0)$ . Finally compute it over a path of your choice going from  $(2, 0)$  to  $(0, 2)$ .