Problem set – Week 9

Parametrizing surfaces

1. Give two parametrizations of the cone $z = \sqrt{x^2 + y^2}$, $0 \leq z \leq r$;
   (a) using cylindrical coordinates,
   (b) using spherical coordinates.
2. Parametrize the cap cut from the sphere $x^2 + y^2 + z^2 = 9$ by the cone $z = \sqrt{x^2 + y^2}$.
3. Parametrize the surface cut from the parabolic cylinder $z = 4 - y^2$ by the planes $x = 0$, $x = 2$, and $z = 0$.
4. Determine the plane tangent to the hemisphere surface
$$\vec{r}(\phi, \theta) = \begin{pmatrix} 4 \sin\phi \cos\theta \\ 4 \sin\phi \sin\theta \\ 4 \cos\phi \end{pmatrix}$$
   for $0 \leq \phi \leq \pi/2$, $0 \leq \theta \leq 2\pi$ at the point $(\sqrt{2}, \sqrt{2}, 2\sqrt{3})$.
5. One obtains a torus of revolution by rotating a circle C with center $(R, 0, 0)$ and radius $r < R$ in the xz-plane about the z-axis. Show that a parametrization of this torus is given by
$$\vec{r}(u, v) = \begin{pmatrix} (R + r\cos u)\cos v \\ (R + r\cos u)\sin v \\ r \sin u \end{pmatrix}$$
   with angles $0 \leq u \leq 2\pi$ and $0 \leq v \leq 2\pi$.