# Solutions – Week 4

### More integration problems, path integrals

1. Compute the following integrals.

$$\text{(a)} \int x \log x \; dx \quad \text{(b)} \int \frac{dx}{x^2\sqrt{x^2+1}} \quad \text{(c)} \int \frac{dx}{x^2(x^2-1)}$$

**Solutions :** (a) $\frac{1}{2}x^2\left(\ln x - \frac{1}{2}\right) + c$, (b) $-x^{-1}\sqrt{x^2+1} + c$, (c) $\frac{1}{x} + \ln\sqrt{\frac{x-1}{x+1}} + c$.

2. Sketch the region enclosed by the line $x = 4$, the curve $y = \sqrt{x}$ and the $x$-axis. Compute its area. Do the same for the region in the first quadrant that is bounded by $y = x^3$ and $y = 4x$.

**Solutions :** 16/3 and 4.

3. Compute the following integrals.

$$\text{(a)} \int_0^\infty \frac{e^{-\sqrt{x}}}{\sqrt{x}} \quad \text{(b)} \int_e^\infty \frac{dx}{x \log x} \quad \text{(c)} \int_0^3 \frac{x \; dx}{(x^2-1)^{2/3}}$$

**Solutions :** (a) 2 ; (b) $\infty$ ; (c) 9/2.

4. For which $x \in (0, 3\pi/2)$ is $f(x) = \int_x^{2x} \frac{\sin t}{t} dt$ a local maximum ?

**Solution :** $\pi/3$.

5. Compute the length of the curve defined by $y = \sqrt{x^3}$ on the interval $0 \leqslant x \leqslant 28$.

**Solution :** 4088/27.

6. Compute the line integral of $x + y^2$ over the segment of the circle $x^2 + y^2 = 4$ going from $(2, 0)$ to $(0, 2)$. Then compute again this line integral but going this time from $(0, 2)$ to $(2, 0)$. Finally compute it over a path of your choice going from $(2, 0)$ to $(0, 2)$.

**Solutions :** Both times $2(2 + \pi)$. In fact, the value of a path integral $\int_C f ds$ does not depend on the orientation or on the parametrization of the path $C$.