Problem set – Week 9

Parametrizing surfaces

1. Give two parametrizations of the cone $z = \sqrt{x^2 + y^2}$, $0 \leq z \leq r$;
   (a) using cylindrical coordinates,
   (b) using spherical coordinates.

Solution : $\vec{r}(z, \theta) = \begin{pmatrix} z\cos\theta \\ z\sin\theta \\ z \end{pmatrix}$, $0 \leq \theta \leq 2\pi$, $0 \leq z \leq r$.

2. Parametrize the cap cut from the sphere $x^2 + y^2 + z^2 = 9$ by the cone $z = \sqrt{x^2 + y^2}$.

Solution : $\vec{r}(\phi, \theta) = \begin{pmatrix} 2\sin\phi\cos\theta \\ 2\sin\phi\sin\theta \\ 2\cos\phi \end{pmatrix}$, $0 \leq \phi \leq \pi/4$, $0 \leq \theta \leq 2\pi$.

3. Parametrize the surface cut from the parabolic cylinder $z = 4 - y^2$ by the planes $x = 0$, $x = 2$, and $z = 0$.

Solution : $\vec{r}(x, y) = \begin{pmatrix} x \\ y \\ \sqrt{4 - y^2} \end{pmatrix}$, $0 \leq x \leq 2$, $-2 \leq y \leq 2$.

4. Determine the plane tangent to the hemisphere surface

$$\vec{r}(\phi, \theta) = \begin{pmatrix} 4\sin\phi\cos\theta \\ 4\sin\phi\sin\theta \\ 4\cos\phi \end{pmatrix}$$

for $0 \leq \phi \leq \pi/2$, $0 \leq \theta \leq 2\pi$ at the point $(\sqrt{2}, \sqrt{2}, 2\sqrt{3})$.

Solution : The equation for the plane is

$$-2x + 2\sqrt{2}y + 4\sqrt{3}z = 28 - 2\sqrt{2}.$$

5. One obtains a torus of revolution by rotating a circle C with center $(R, 0, 0)$ and radius $r < R$ in the xz-plane about the z-axis. Show that a parametrization of this torus is given by

$$\vec{r}(u, v) = \begin{pmatrix} (R + r\cos u)\cos v \\ (R + r\cos u)\sin v \\ r\sin u \end{pmatrix}$$

with angles $0 \leq u \leq 2\pi$ and $0 \leq v \leq 2\pi$.