Midterm exam

Num. CSE, D-INFK/D-MATH

HS 2015

Prof. R. Hiptmair

Name		Grade
Surname		
Department		
Legi Nr.		
Date	23.10.2015	

1	2	3	4	5	6	Total
8	8	8	8	10	12	54

- **Only writing material and Legi on the table**
- Keep mobile phones, tablets, smartwatches, etc. **turned off** in your bag
- Fill in this cover sheet first
- Turn the cover sheet only when instructed to do so
- Then, put your name and Legi Nr. on each page
- Carefully read the rules for the multiple-choice problems
- Do not write with red/green colour or with pencil
- **Write your solution in the predefined tick boxes**
- **Before handing-in the sheets, make sure you wrote the name on each sheet**

- **Duration: 30 min**
- Additional material: none

Good luck!

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Rules of multiple-choice (MC):

- *(Clearly) put a cross in the circle with the answer you deem correct.*
- *Motivation for the choices is* ***not*** *necessary. Remarks and computations have* ***no*** *influence on the total number of points.*
- ***Wrong answers give negative points.***
- *Mutually exclusive answers are weighted such that the expectation of a random choice is zero.*
- *The total points for a single MC-problem is the maximum between the points achieved and zero.*
- *Mutually exclusive answers are highlighted in the same box.*
- *Any unclear marking will be considered an error.*

1. *Cancellation errors* [8 P.]

   Which expressions are likely to be affected by cancellation errors for some of the xs in the indicated ranges?

   (a) `y = (sin(x) + x)/x` for x in $[-0.1, 0.1]$

○ Affected by cancellation.	☒ Not affected by cancellation.

   -2 +2

   (b) `z = exp(x), y = z - 1/z` for x in $[-1, 1]$

☒ Affected by cancellation.	○ Not affected by cancellation.

   +2 -2

   (c) `y = sqrt(x + 1) - sqrt(x)` for x in $[0.9, 1.1]$

○ Affected by cancellation.	☒ Not affected by cancellation.

   -2 +2

   (d) `y = log(x) - log(x - 1)` for x in $[2, +\infty)$

☒ Affected by cancellation.	○ Not affected by cancellation.

   +2 -2

Scratch space (not evaluated):

2. *The Newton iteration* [8 P.]

Take $a \in \mathbb{R}$. Let $F\colon \mathbb{R} \to \mathbb{R}$ be defined by $F(x) = xe^x - a$ for $x \in \mathbb{R}$. Establish which of the following iterations of the Newton method are correct for solving $F(x) = 0$.

(a) $x^{(k+1)} = \frac{x^{(k)}}{x^{(k)}+1}\left(x^{(k)} + \frac{ae^{-x^{(k)}}}{x^{(k)}}\right)$.

☒ Correct iteration.	○ Incorrect iteration.
+2	−2

(b) $x^{(k+1)} = x^{(k)} - e^{x^{(k)}}\left(a - x^{(k)}e^{x^{(k)}}\right)\left(x^{(k)} + 1\right)$.

○ Correct iteration.	☒ Incorrect iteration.
−2	+2

(c) $x^{(k+1)} = x^{(k)} + \frac{a - x^{(k)}e^{x^{(k)}}}{x^{(k)}e^{x^{(k)}} + e^{x^{(k)}}}$.

☒ Correct iteration.	○ Incorrect iteration.
+2	−2

(d) $x^{(k+1)} = x^{(k)} - \frac{x^{(k)}e^{x^{(k)}} + e^{x^{(k)}}}{x^{(k)}e^{x^{(k)}} - a}$.

○ Correct iteration.	☒ Incorrect iteration.
−2	+2

Scratch space (not evaluated):

3. *Numerical stability* [8 P.]

Let $\tilde{F}: X \to Y$ be an algorithm for the problem $F: X \to Y$. Let $w(x)$ denote the computational effort required for evaluating $\tilde{F}$ with input x. Establish which of the following is the correct definition of numerical stability. Here eps represents the machine precision.

We say that $\tilde{F}$ is *numerically stable* if there exists $C \approx 1$ such that

(a) $\forall x \in X. \quad \|F(x) - \tilde{F}(x)\|_Y \leq Cw(x)\mathsf{eps}\|\tilde{F}(x)\|_Y.$

(b) $\forall x \in X. \quad \exists \tilde{x} \in X. \quad \|\tilde{x} - x\|_X \leq Cw(x)\mathsf{eps}\|x\|_X \;\wedge\; \tilde{F}(\tilde{x}) = F(x).$

(c) $\forall x \in X. \quad \exists \tilde{x} \in X. \quad \|F(x) - \tilde{F}(\tilde{x})\|_Y \leq Cw(x)\mathsf{eps}\|F(x)\|_Y.$

(d) $\forall x \in X. \quad \exists \tilde{x} \in X. \quad \|x - \tilde{x}\|_X \leq Cw(x)\mathsf{eps}\|x\|_X \;\wedge\; \tilde{F}(x) = F(\tilde{x}).$

Which one is the correct definition?

○ (a)	○ (b)	○ (c)	☒ (d)
- 8/3	- 8/3	- 8/3	+ 8

Scratch space (not evaluated):

4. *Fixed point iteration* [8 P.]

A fixed point iteration for solving $F(x) = 0$, $F = [F_1, F_2]^T : D \subset \mathbb{R}^2 \to \mathbb{R}^2$ is known to be **affine invariant**. Which of the following conclusions are true?

(a) The iterates for F and $\tilde{F} = [F_2, F_1]^T$ agree if the initial guesses are the same.

☒ True.	○ False.
+2	-2

(b) When applied to $\tilde{F} = [F_1 + F_2, 0]^T$, the method produces the same iterates.

○ True.	☒ False.
-2	+2

(c) The method produces the same iterates for F and for $\tilde{F}$ defined by $\tilde{F}(x_1, x_2) = F(x_2, x_1)$.

○ True.	☒ False.
-2	+2

(d) Let $x^{(k)}$ and $\tilde{x}^{(k)}$ be two different sequences generated by the fixed point method applied to F. If $\tilde{x}^{(0)} = Ax^{(0)}$ for some regular matrix $A \in \mathbb{R}^{2,2}$, then $\tilde{x}^{(k)} = Ax^{(k)}$ for every k.

○ True.	☒ False.
-2	+2

Scratch space (not evaluated):

5. *C++ code* [10 P.]

Consider the C++ and Eigen code:

```
#include <Eigen/Dense>
using Eigen;
void fn(const Matrix<double, Dynamic, Dynamic, RowMajor> &A,
        const VectorXd &v, VectorXd &w) {
  const double *a = A.data(); // ptr. to linear data of A
  const std::size_t n = A.rows();
  assert( ( n == v.size() ) && ( n == w.size() ));
  for(std::size_t i = 0; i < n; ++i) {
    a += i;
    w(i) = 0.;
    for(std::size_t j = i; j < n; ++j) {
      w(i) += (*a)*v(j);
      a++;
    }
  }
}
```

For each of the following lines of Matlab code, decide whether it is algebraically equivalent to the function fn or not.
HINT: The functions `triu(A)` and `tril(A)` return the upper and lower triangular parts of `A`, respectively.
HINT: Have a look at the figure in the next page for an picture hint.

Algebraically equivalent?

(a) `w = (A+A')*v;` — ○ yes. ☒ no.
-5/3 +5/3

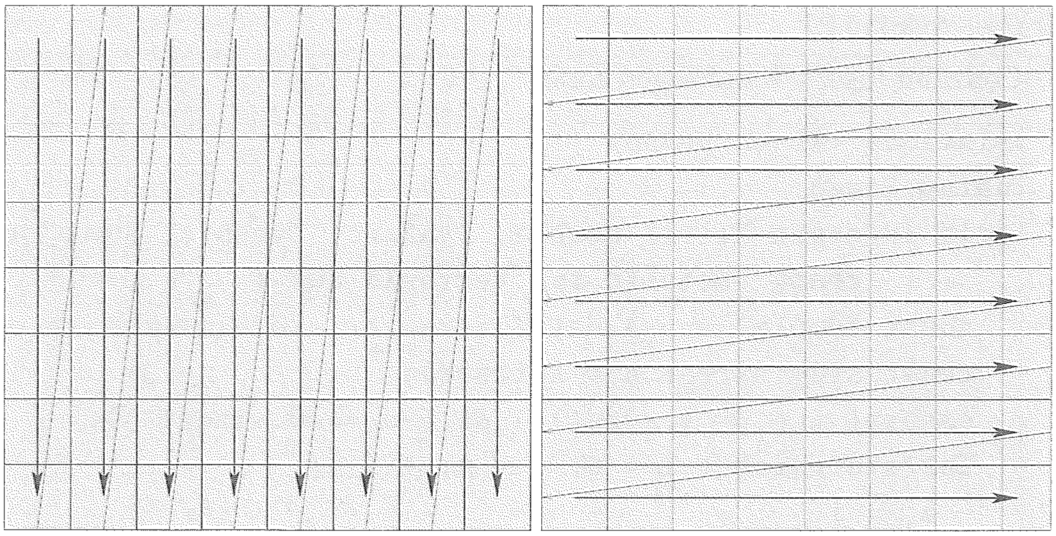
(b) `w = triu(A)*v;` — ☒ yes. ○ no.
+5/3 -5/3

(c) `w = triu(A')*v;` — ○ yes. ☒ no.
-5/3 +5/3

(d) `w = tril(A)*v;` — ○ yes. ☒ no.
-5/3 +5/3

(e) `w = tril(A')*v;` — ○ yes. ☒ no.
-5/3 +5/3

(f) `w = A*v;` — ○ yes. ☒ no.
-5/3 +5/3



Column major Row major

Scratch space (not evaluated):

6. *QR-decomposition* [12 P.]

Consider the MATLAB code:

```
  function [x,r] = solveandmore(A,b)
2     [n,m] = size(A);
      if(n ~= m), error('A must be square'); end
4     [Q,R] = qr(A)
      r = min(diag(R));
6     x = A\b;
  end
```

For each of the following lines of code, decide whether it is algebraically equivalent to line 6 above or not. Provide the asymptotic complexity of the substitute line, if `A` is a generic dense square matrix.

HINT: Matlab's \ can detect triangular system matrices.

(a) `x = R\(Q\b);`

+1	-1
☒ algebraically equivalent.	○ **not** algebraically equivalent.

○ $O(n)$.	○ $O(n^2)$.	☒ $O(n^3)$.
-1/2	-1/2	+1

(b) `x = R\(Q'*b);`

+1	-1
☒ algebraically equivalent.	○ **not** algebraically equivalent.

○ $O(n)$.	☒ $O(n^2)$.	○ $O(n^3)$.
-1/2	+1	-1/2

(c) `x = (Q*R)\b;`

+1	-1
☒ algebraically equivalent.	○ **not** algebraically equivalent.

○ $O(n)$.	○ $O(n^2)$.	☒ $O(n^3)$.
-1/2	-1/2	+1

(d) `x = Q\(R\b);`

-1	+1
○ algebraically equivalent.	☒ **not** algebraically equivalent.

○ $O(n)$.	○ $O(n^2)$.	☒ $O(n^3)$.
-1/2	-1/2	+1

(e) `x = (Q\R)\b;`

-1	+1
○ algebraically equivalent.	☒ **not** algebraically equivalent.

○ $O(n)$.	○ $O(n^2)$.	☒ $O(n^3)$.
-1/2	-1/2	+1

(f) `x = Q'*(R\b);`

-1	+1
○ algebraically equivalent.	☒ **not** algebraically equivalent.

○ $O(n)$.	☒ $O(n^2)$.	○ $O(n^3)$.
-1/2	+1	-1/2

Scratch space (not evaluated):