Schnellübung 7

1. Berechnen Sie die folgenden uneigentlichen Integrale, sofern diese existieren.

a) $\displaystyle\int_2^\infty \frac{dx}{x\ln x}$,

b) $\displaystyle\int_2^\infty \frac{dx}{x(\ln x)^2}$,

c) $\displaystyle\int_0^\infty \frac{x\,dx}{\sqrt{1+x^4}}$.

2. Zeigen Sie, dass für $m, n \in \mathbb{N}$ gilt:

$$\int_0^1 x^m (\ln x)^n \, dx = (-1)^n \frac{n!}{(m+1)^{n+1}}$$

3. Entscheiden Sie, welche der folgenden uneigentlichen Integrale konvergieren:

a) $\displaystyle\int_{-\infty}^{+\infty} \frac{dx}{\cosh x}$,

b) $\displaystyle\int_0^{\frac{\pi}{2}} \frac{\tan x}{x}\, dx$,

c) $\displaystyle\int_0^1 \frac{dx}{\sqrt{x-x^3}}\, dx$,

d) $\displaystyle\int_0^1 \frac{dx}{x^4-x^5}$.

Bitte wenden!

English version

1. Compute the following improper integrals, provided that they exist.

a) $\displaystyle\int_2^{\infty} \frac{dx}{x \ln x}$,

b) $\displaystyle\int_2^{\infty} \frac{dx}{x(\ln x)^2}$,

c) $\displaystyle\int_0^{\infty} \frac{x\,dx}{\sqrt{1+x^4}}$.

2. Show that for all $m, n \in \mathbb{N}$ the following identity holds true:

$$\int\limits_0^1 x^m (\ln x)^n \, dx = (-1)^n \frac{n!}{(m+1)^{n+1}}$$

3. Determine whether the following improper integrals converge:

a) $\displaystyle\int_{-\infty}^{+\infty} \frac{dx}{\cosh x}$,

b) $\displaystyle\int_0^{\frac{\pi}{2}} \frac{\tan x}{x}\, dx$,

c) $\displaystyle\int_0^1 \frac{dx}{\sqrt{x - x^3}}\, dx$,

d) $\displaystyle\int_0^1 \frac{dx}{x^4 - x^5}$.