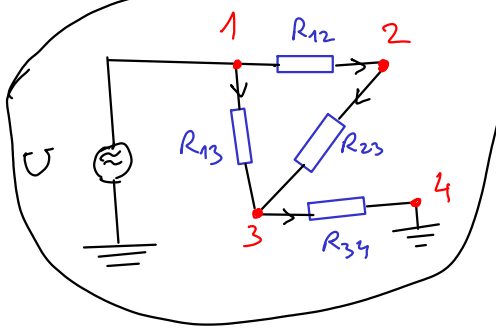
Wir betrachten den Schaltplan:



Gegeben die Widerstände R_{kj} und die Spannung U, berechne die Knotenspannungen U_k

Ohm: $I_{kj} = \frac{U_j - U_k}{R_{kj}}$

Kirchhoff:

1: $U_1 = U$

2: $I_{12} - I_{23} = 0$ $\Rightarrow \frac{U_2 - U_1}{R_{12}} - \frac{U_3 - U_2}{R_{23}} = 0 \Rightarrow \left(\frac{1}{R_{12}} + \frac{1}{R_{23}}\right) U_2 - \frac{1}{R_{23}} U_3 = \frac{1}{R_{12}} U$

3: $I_{13} + I_{23} - I_{34} = 0 \Rightarrow \frac{U_3 - U_1}{R_{13}} + \frac{U_3 - U_2}{R_{23}} - \frac{U_4 - U_3}{R_{34}} = 0 \Rightarrow -\frac{1}{R_{23}} U_2 + \left(\frac{1}{R_{13}} + \frac{1}{R_{23}} + \frac{1}{R_{34}}\right) U_3 = \frac{U}{R_{13}}$

4: $U_4 = 0$

$$\Rightarrow \begin{cases} \left(\frac{1}{R_{12}} + \frac{1}{R_{23}}\right) U_2 + \left(-\frac{1}{R_{23}}\right) U_3 = \frac{1}{R_{12}} U \\ -\frac{1}{R_{23}} U_2 + \left(\frac{1}{R_{13}} + \frac{1}{R_{23}} + \frac{1}{R_{34}}\right) U_3 = \frac{1}{R_{13}} U \end{cases}$$

schreibe:

$$\begin{bmatrix} \frac{1}{R_{12}} + \frac{1}{R_{23}} & -\frac{1}{R_{23}} \\ -\frac{1}{R_{23}} & \frac{1}{R_{13}} + \frac{1}{R_{23}} + \frac{1}{R_{34}} \end{bmatrix} \begin{bmatrix} U_2 \\ U_3 \end{bmatrix} = \begin{bmatrix} \frac{1}{R_{12}} \\ \frac{1}{R_{13}} \end{bmatrix} U$$

lineares Gleichungssystem

$R_{12} = R_{23} = R_{34} = R_{13} = 1 \Rightarrow$

$$\begin{cases} 2U_2 - U_3 = U \\ -U_2 + 3U_3 = U \end{cases}$$

$\left| \cdot \frac{1}{2} \right) \oplus \Rightarrow 0 + \left(-\frac{1}{2} + 3\right) U_3 = \frac{3}{2} U \Rightarrow \begin{cases} 2U_2 - U_3 = U \\ \frac{5}{2} U_3 = \frac{3}{2} U \end{cases} \left| \cdot \frac{2}{5} \right. \Rightarrow \begin{cases} 2U_2 - U_3 = U \\ U_3 = \frac{3}{5} U \end{cases} \Rightarrow$

$\Rightarrow 2U_2 - \frac{3}{5} U = U \Rightarrow 2U_2 = \frac{8}{5} U \Rightarrow U_2 = \frac{4}{5} U$

Somit $U_2 = \frac{4}{5} U$, $U_3 = \frac{3}{5} U$ oder $\begin{bmatrix} U_2 \\ U_3 \end{bmatrix} = \begin{bmatrix} 0.8 \\ 0.6 \end{bmatrix} U$