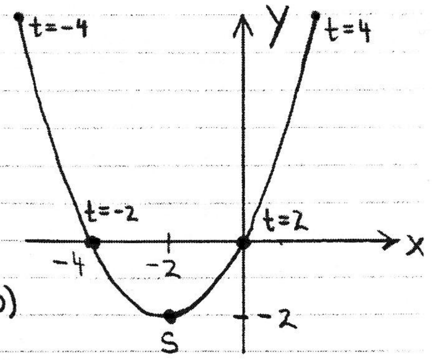
Übungsserie 1, HS 2015, Seite 1



① (a) $\begin{pmatrix}0\\y\end{pmatrix} \overset{\text{soll}}{=} \begin{pmatrix}t-2\\0.5t^2-2\end{pmatrix} \rightsquigarrow t_0=2$, in $y(t)$: $y_0 = 0.5\cdot 4 - 2 = 0 \rightsquigarrow (0,0)$

$\begin{pmatrix}x\\0\end{pmatrix} \overset{\text{soll}}{=} \begin{pmatrix}t-2\\0.5t^2-2\end{pmatrix} \rightsquigarrow$ $0.5t^2 = 2$, $t^2 = 4$, $t_1 = 2$, $t_2 = -2$ in $x(t)$: $x_1 = 2-2 = 0 \rightsquigarrow (0,0)$, $x_2 = -2-2 = -4 \rightsquigarrow (-4,0)$

(b) $y(t) = \underbrace{0.5t^2}_{\geq 0} - 2 \geq -2$ und $= -2$ nur für $t=0$, in $x(t)$: $x_s = -2$, $y(t)$: $y_s = -2$ $\rightsquigarrow S = (-2,-2)$

(c) $x(t) = t-2 \rightsquigarrow t = x+2$ in $y(t) = 0.5t^2 - 2 = 0.5(x+2)^2 - 2 = 0.5(x^2+4x+4) - 2$

$y = 0.5x^2 + 2x$ (Parabel mit Scheitel S)

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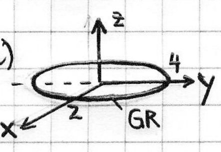
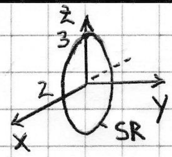
② (a)



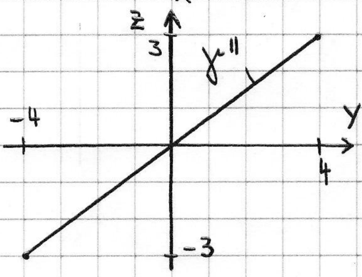
Ellipse mit $a=2, b=4$ $\begin{pmatrix}a\cos t\\b\sin t\\\dots\end{pmatrix} = \begin{pmatrix}2\cos t\\4\sin t\\\dots\end{pmatrix}$



Ellipse mit 2,3 $\begin{pmatrix}2\cos t\\\dots\\3\sin t\end{pmatrix}$

$\gamma: [0, 2\pi] \to \mathbb{R}^3,\ t \mapsto \vec{r}(t) = \begin{pmatrix}2\cos t\\4\sin t\\3\sin t\end{pmatrix}$

(b) $y = 4\sin t \rightsquigarrow \sin t = \frac{1}{4}y$
$z = 3\sin t$ in z: $z = \frac{3}{4}y$



(Räumliche Darstellung ähnlich wie Figur 1.8 im Skript)

(c) $\begin{pmatrix}\sqrt{3}\\y\\z\end{pmatrix} \overset{\text{soll}}{=} \begin{pmatrix}2\cos t\\4\sin t\\3\sin t\end{pmatrix} \rightsquigarrow \cos t = \sqrt{3}/2$, $t_1 = \frac{\pi}{6}$, $t_2 = \frac{11\pi}{6}$

in $y(t) = 4\sin(t_1) = 2$ bzw. $4\sin(t_2) = -2$
in $z(t) = 3\sin(t_1) = 1.5$ bzw. $= -1.5$

Durchstosspkte: $(\sqrt{3}, 2, 1.5)$ und $(\sqrt{3}, -2, -1.5)$

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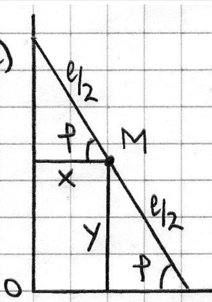
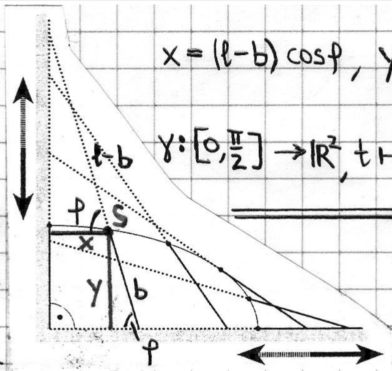
③ (a)



$\frac{x}{\ell/2} = \cos\varphi$, $\frac{y}{\ell/2} = \sin\varphi$

$x = \ell/2 \cos\varphi$, $y = \ell/2 \sin\varphi$

$\gamma: [0, \frac{\pi}{2}] \to \mathbb{R}^2,\ t \mapsto \vec{r}(\varphi) = \begin{pmatrix}\ell/2 \cos\varphi\\\ell/2 \sin\varphi\end{pmatrix}$



$x = (\ell - b)\cos\varphi$, $y = b\cdot\sin\varphi$

$\gamma: [0, \frac{\pi}{2}] \to \mathbb{R}^2,\ t \mapsto \vec{r}(\varphi) = \begin{pmatrix}(\ell-b)\cos\varphi\\b\sin\varphi\end{pmatrix}$

(c) Bei (a): (Viertel) Kreis mit Mittelpunkt O, Radius $\ell/2$

Bei (b): (Viertel) Ellipse mit Mittelpunkt O und Halbachsen $a = \ell - b$, b

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④ Zur Erinnerung die Regeln: ① $(x^n)' = nx^{n-1}$, speziell: $(x^1)' = 1\cdot x^0 = 1$, $(\text{Zahl})' = 0$ ② $(f+g)' = f' + g'$ ③ $(\text{Zahl}\cdot f)' = \text{Zahl}\cdot f'$ ④ $(\cos x)' = -\sin x$, $(\sin x)' = \cos x$ ⑤ $(f\cdot g)' = f'\cdot g + f g'$ ⑥ $[f(g(x))]' = f'(g(x))\,\underbrace{g'(x)}_{\text{innere Abl.}}$

(a) $x'(t) = (t-1)' \overset{②}{=} (t)' - (1)' \overset{①}{=} \underline{1}$ (b) $y'(t) = (2t-1)' \overset{②}{=} (2t)' - (1)' \overset{③}{=} 2(t)' - (1)' \overset{①}{=} \underline{2}$

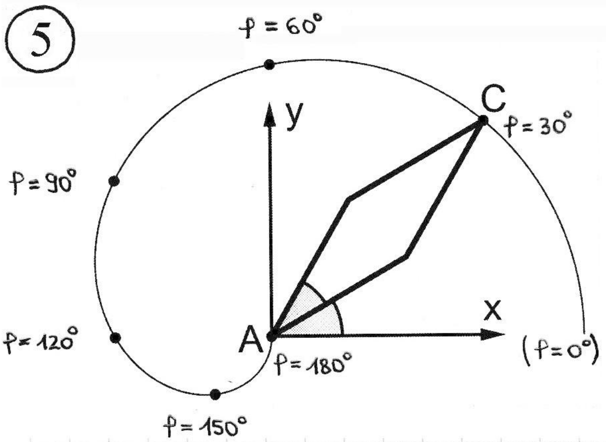
(c) $y'(t) = \Big(R\underbrace{\cos}_{f(\cdot)}\big(\underbrace{\tfrac{2\pi}{60s}t}_{g(t)}\big)\Big)' \overset{④⑥}{=} R\left(-\sin\left(\tfrac{2\pi}{60s}t\right)\right)\cdot\underbrace{\left(\tfrac{2\pi}{60s}\cdot t\right)'}_{\text{innere Abl}} \overset{①}{=} -R\sin\left(\tfrac{2\pi}{60s}t\right)\tfrac{2\pi}{60s} = \underline{\underline{-\tfrac{2\pi R}{60s}\sin\left(\tfrac{2\pi}{60s}t\right)}}$

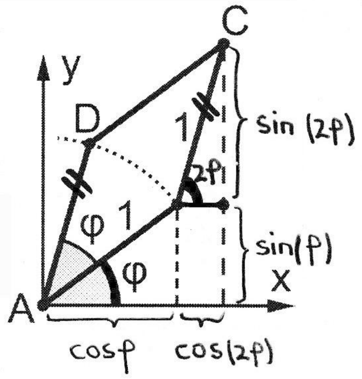
(d) $x'(t) = (a\underbrace{t\cos t}_{f\cdot g})' \overset{③⑤}{=} a[\underbrace{(t)'\cos t}_{f'\cdot g} + \underbrace{t(\cos t)'}_{f\cdot g'}] \overset{①④}{=} \underline{\underline{a[\cos t - t\sin t]}}$ (e) $y'(t) = (t\underbrace{R\sin\varphi}_{\text{Zahl}})' \overset{③}{=} R\sin\varphi\,(t)' \overset{①}{=} \underline{\underline{R\sin\varphi}}$

(f) $y'(\varphi) = (tR\sin\varphi)' \overset{③}{=} tR(\sin\varphi)' \overset{④}{=} \underline{\underline{tR\cos\varphi}}$ (g) $z'(\varphi) = \left(\tfrac{h}{2\pi}\varphi\right)' \overset{③}{=} \tfrac{h}{2\pi}(\varphi)' \overset{①}{=} \underline{\underline{\tfrac{h}{2\pi}}}$ (h) $z'(t) = \Big(\underbrace{\tfrac{h}{2\pi}\varphi}_{\text{Zahl}}\Big)' \overset{①}{=} \underline{\underline{0}}$

(i) $y'(\varphi) = (\underbrace{e^\varphi\sin\varphi}_{f\cdot g})' \overset{⑤}{=} \underbrace{(e^\varphi)'\sin\varphi}_{f'\cdot g} + \underbrace{e^\varphi(\sin\varphi)'}_{f\cdot g'} \overset{④}{=} \underline{\underline{e^\varphi\sin\varphi + e^\varphi\cos\varphi}}$ (da $(e^\varphi)' = e^\varphi$)

(5)

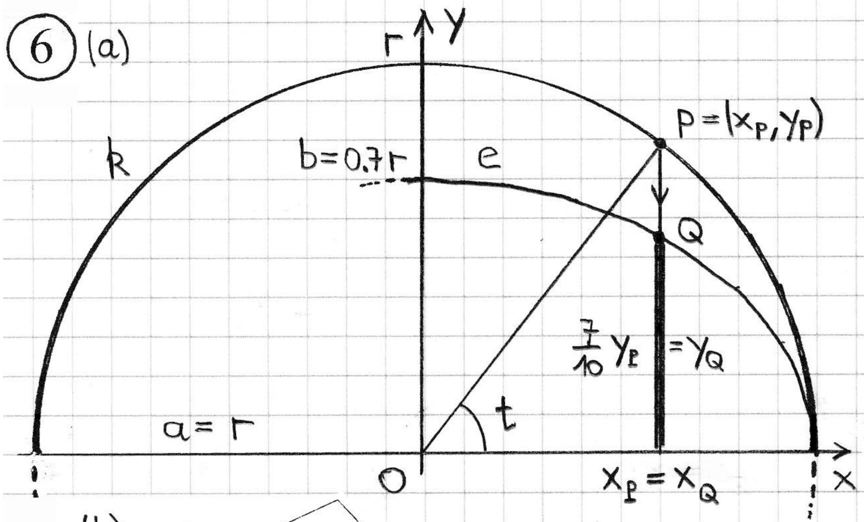




$$x(\varphi) = \cos\varphi + \cos(2\varphi)$$
$$y(\varphi) = \sin\varphi + \sin(2\varphi) \qquad (0 \leq \varphi \leq 180°)$$

$$\varphi \mapsto \vec{r}(\varphi) = \begin{pmatrix} \cos\varphi + \cos(2\varphi) \\ \sin\varphi + \sin(2\varphi) \end{pmatrix}$$

(6) (a)



Für P auf k gilt: $\vec{OP} = \begin{pmatrix} r\cos t \\ r\sin t \end{pmatrix}$

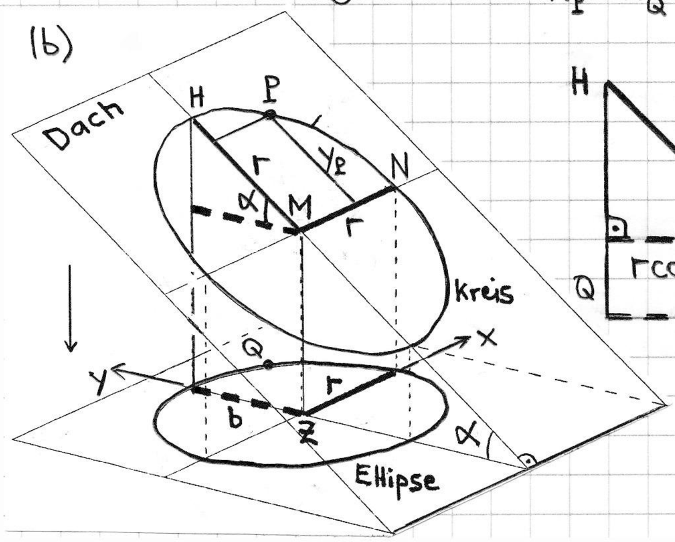
Stauchung: $y_Q = \frac{7}{10} y_P$

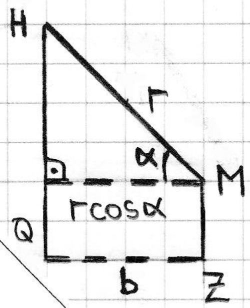
Für Q auf e gilt: $\vec{OQ} = \begin{pmatrix} r\cos t \\ 0.7r\sin t \end{pmatrix}$

$\rightsquigarrow$ Ellipse mit $a = r$ und $b = 0.7r$

(Bedeutung von Parameter t in e: t ist der Drehwinkel in der Parameterdrst. des gestauchten Kreises)

(b)





$|MN|$ und Linien parallel zu MN werden nicht verkürzt $\rightsquigarrow$ $x_Q = x_P$, $a = r$

$|HM|$ und Linien parallel zu HM (Falllinien) werden verkürzt (gestaucht) $\rightsquigarrow$ $y_Q = y_P \cos\alpha = \frac{\sqrt{2}}{2} y_P$

$b = r \cdot \cos\alpha = \frac{\sqrt{2}}{2} r$

Lichtfleck ist ein mit dem Faktor $\cos\alpha$ gestauchter Kreis $\rightsquigarrow$ Ellipse mit $a = r$, $b = r\cos\alpha$