Modelling of trades-throughs in a limit order book with Hawkes processes

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Abstract

Recent contributions have emphasized that Hawkes processes exhibit interesting features for financial modelling. For example, these self- and mutually exciting point processes can model arrival time of orders in an order book (Large (2007); Muni Toke (2011)), or explain the Epps effect in a microstructure toy model (Bacry et al. (2011)). A comprehensive framework can be derived (Bowhsher (2007)).

Here, we are interested in modelling trades-through, i.e. transactions that reach at least the second level of limit orders in an order book. Trades-through are very important in price formation and microstructure. Since trades usually minimize their market-impact by splitting their orders accroding to the liquidity available in the order book, trades-through may contain more information. They may laso reach gaps in orders books, which is crucial in price dynamics.

In a first part, we give basic statistical facts on trades-through, focusing on their arrival times and clustering properties. Our second part is a general introduction to Hawkes processes. In a third part, using tick-by-tick data on Euronext-traded stocks, we show that a simple bi-dimensional Hawkes process fits nicely our empirical data of trades-through. We show that the cross-influence of bid and ask trades-through is weak. Following Bowsher (2007), we improve the statistical performance of our maximum likelihood calibrations by enhancing the stationary model using deterministic time-dependent base intensity.

Keywords : Hawkes processes, limit order book, trades-through, high-frequency trading, microstructure.