Zbl 080.03305
Erdős, Paul
Sur certaines séries à valeur irrationnelle.
On certain series with irrational values (In French)
Enseign. Math., II. Ser. 4, 93-100 (1958). [0013-8584]

Let p_n be the nth prime. The author states that he has proved the irrationality of the series $\sum_{n=1}^{\infty} \frac{p_n^k}{n!}$ $(k = 1, 2, 3, ...)$, and he gives the proof for $k = 1$. It is based on the fact that the numbers $p_n/n - [p_n/n]$ lie dense in $(0, 1)$. More generally, let $\{q_n\}$ be a sequence of integers such that $1 < q_1 \leq q_2 \leq q_3 \leq \cdots$, $q_n > cn/(\log n)^k$ ($c > 0$, $k > 0$ constants). Then he shows that

$$\sum_{n=1}^{\infty} \frac{p_n}{q_1 q_2 \cdots q_n}$$

is irrational unless $q_n = qp_n + 1$ for $n \geq n_0$, where $q \geq 1$ is an integer. This result can be further improved.

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Classification:
11J72 Irrationality

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