Zbl 106.27702
Erdős, Pál
An inequality for the maximum of trigonometric polynomials (In English)
Ann. Pol. Math. 12, 151-154 (1962). [0066-2216]

Let

$$f_n(\theta) = \sum_{k=1}^{n} (a_k \cos k\theta + b_k \sin k\theta)$$

be a trigonometric polynomial with real coefficients. Put $M = \max_{0 \le \theta < 2a} |f_n(\theta)|$. The author conjectures that there exists an absolute constant $c > 0$ such that

$$(1) \quad M \ge \frac{1+c}{\sqrt{2}} \left\{ \sum_{k=1}^{n} (a_k^2 + b_k^2) \right\}^{1/2},$$

with $c \le \sqrt{2} - 1$.
Theorem: Assume that $\max_{1 \le k \le n} (\max |a_k|, |b_k|) = 1$ and that $\sum_{k=1}^{n} (a_k^2 + b_k^2) = An$. Then there exists a $c = c_A > 0$ depending on A for which $\lim_{A \to 0} c_A = 0$ and

$$M > \frac{1+c_A}{\sqrt{2}} \left\{ \sum_{k=1}^{n} (a_k^2 + b_k^2) \right\}^{1/2}. \tag{2}$$

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Classification:
42A05 Trigonometric polynomials