
Zbl 125.08602
Erdős, Pál
On some applications of probability to analysis and number theory (In English)
J. Lond. Math. Soc. 39, 692-696 (1964).

The author discusses applications of probability theory for five problems of analysis, among them the following:
1. For what sequence of integers $n_1 < n_2 < \cdots$ does there exist a power series $\sum_{k=1}^{\infty} a_k z^{n_k}$ converging uniformly in $|z| \leq 1$ but for which $\sum_{k=1}^{\infty} |a_k| = \infty$?
2. It is known that $f_t(z) = \sum_{k=0}^{\infty} \varepsilon_k a_k z^k$ where $\varepsilon_k = \pm 1$, $t = \sum_{k=1}^{\infty} \frac{1+\varepsilon_k}{2^{k+1}}$ and $\sum_{k=1}^{\infty} |a_k|^2 = \infty$, diverges almost everywhere on the unit circle if $|a_k| \geq c_k$ where $c_k > 0$ is a monotone sequence of numbers tending to zero so that

$$\limsup_{k=\infty} \left[\left(\sum_{j=1}^{k} c_j^2 \right) / \log(1/c_k) \right] > 0.$$

If this does not hold, is there a sequence $\{a_k\}$ such that $|a_k| \geq c_k$, for which $f_t(z)$ has at least one point of convergence for all t?
Some unpublished probabilistic methods in number theory conclude the paper.

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Classification:
11N25 Distribution of integers with specified multiplicative constraints
11K99 Probabilistic theory
30B10 Power series (one complex variable)