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On the mean value of nonnegative multiplicative number-theoretical functions (In English)

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Let $g(n)$ be a nonnegative and strongly multiplicative function [i.e. $g(mn) = g(m)g(n)$ for $(m,n) = 1$ and $g(p^k) = g(p)$ for prime p and $k = 1, 2, ...$], and let $M(g) = \lim_{N\to\infty} \frac{1}{N} \sum_{n\le N} g(n)$, if the limit exists. The authors consider the following conditions: (i) the series $\sum_p \frac{g(p)-1}{p}$ is convergent, (ii) the series $\sum_p \frac{[g(p)]^2}{p^2}$ is convergent, (iii) for every positive ε, $\sum_{n\le p\le N(1+\varepsilon)} \frac{g(p)\log p}{p} \ge \delta(\varepsilon)$ for $N \ge N(\varepsilon)$ with suitable δ ($\varepsilon > 0$) and $N(\varepsilon)$, and prove (Theorem 2) that (i), (ii) and (iii) imply

$$M(g) = \prod_p \left[1 + \frac{g(p)-1}{p}\right].$$

If (i) and (iii) are satisfied, but (ii) is not, then $M(g)$ exists and is equal to zero (Theorem 6). A similar result is then deduced for general multiplicative functions, and finally some counterexamples are given.

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Classification:
11N37 Asymptotic results on arithmetic functions