
Zbl 134.01602

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Some remarks concerning our paper 'On the structure of set-mappings'. Non-existence of a two-valued σ-measure for the first uncountable inaccessible cardinal (In English)

Acta Math. Acad. Sci. Hung. 13, 223-226 (1962). [0001-5954]

A cardinal m has property P_3 if every two-valued measure μ defined on the power set of a set S of power m is identically zero, provided that $\mu(x)$ is m-additive and $\mu(\{x\}) = 0$ for all $x \in S$. [Cf. *Erdős* and *Tarski*, Essays Foundations Math., dedicat, to A. A. Fraenkel on his 70th Anniversary, 50-82 (1962; Zbl 212.32502).] In the authors previous paper (Zbl 102.28401), they proved:
(i) If $m > \aleph_0$ is strongly inaccessible and does not have property P_3, then $m \to (m)^{<\aleph_0}$.
(ii) $m \not\to (\aleph_0)^{\aleph_0}$ for every $m < t_1$, where t_1 is the first uncountable strongly inaccessible ordinal. The partition notation $m \to (n)^{\aleph_0}$ comes from *P.Erdős* and *R.Rado* (Zbl 071.05105).
They now derive from (ii) the additional result (iii): $t_1 \not\to (\aleph_1)^{\aleph_0}$. From (i) and (iii) it follows that t_1 has property P_3, which had already been proved by Tarski and by Keisler. The authors state the following generalization of (iii):
(iv) If n is either $\aleph_0$ or not strongly inaccessible and t_ξ is the least strongly inaccessible ordinal $> n$, then $t_\xi \not\to (n^+)^{<\aleph_0}$. If $t_0, t_1, ..., t_\xi, ...$ is an enumeration of all strongly inaccessible cardinals, then (i) and (iv) imply that, if $\xi < t_\xi$ has P_3. Among the unsolved problems mentioned, two of the simplest are: $t_{\xi_0} \not\to (t_{\xi_0})^{\aleph_0}$ (where ξ_0 is the least ordinal for which $\xi_0 = t_\xi$), and $t_1 \to (\aleph_0)^{\aleph_0}$.

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Classification:
05D10 Ramsey theory
03E55 Large cardinals
04A20 Combinatorial set theory
05E05 Symmetric functions
04A10 Ordinal and cardinal numbers; generalizations