Zbl 151.03502
Erdős, Pál; Sarközy, A.; Szemeredi, E.
On the solvability of the equations $[a_i, a_j] = a_r$ *and* $(a'_i, a'_j) = a'_r$ *in sequences of positive density* (In English)
J. Math. Anal. Appl. 15, 60-64 (1966). [0022-247X]

The authors obtain the following results.
1) Let $a_1 < a_2 < \cdots$ be an infinite sequence of integers for which there are infinitely many integers $n_1 < n_2 < \cdots$ satisfying

$$\sum_{a_i < n_k} \frac{1}{a_i} > c_1 \frac{\log n_k}{(\log\log n_k)^{1/2}}.$$

Then the equations $(a'_i, a'_j) = a'_r, [a_i, a_j] = a_r$ have infinitely many solutions. The symbol (a_i, a_j) denotes the greatest common divisor and $[a_i, a_j]$ denotes the least common multiple of a_i and a_j.
2) Let $a_1 < a_2 < \cdots$ be an infinite sequence of integers for which there are infinitely many integers $n_1 < n_2 < \cdots$ satisfying

$$\sum_{a_i < n_k} \frac{1}{a_i} > c_2 \frac{\log n_k}{(\log\log n_k)^{1/4}}.$$

Then there are infinitely many quadruplets of distincts integers a_i, a_j, a_r, a_s satisfying $(a_i, a_j) = a_r$, $[a_i, a_j] = a_s$, c_1 and c_2 denote suitable positive constants.

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Classification:
11B83 Special sequences of integers and polynomials