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Erdős, Pál
On a combinatorial problem. III (In English)
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[Part I in Nordisk. Mat. Tidskr. 11, 5-10 (1963; Zbl 116.01104)]
E.W.Miller, in C. R. Soc. Sci. Varsovie 30, 31-38 (1937; Zbl 017.30003), defines a family of sets $\{A\}$ th have property B if there exists a set S which meets all the sets A_k and contains none of them. The author and *A.Hajnal* [Acta Math. Acad. Sci. Hungar. 12, 87-123 (1961; Zbl 201.32801)] define $m(n)$ as the smallest integer for which there is a family of $m(n)$ sets, each with cardinality n, which do not have property B. In Part II [ibid. 15, 445-447 (1964)], the author had found bounds for $m(n)$. In this paper he considers the function $m_N(n)$ which is the smallest integer for which there are $m_N(n)$ sets A_k each with cardinality n which are all subsets of a set S, $|S| = N$, and which do not have property B. It is shown that if $N = (c + o(1))n$ then

$$\lim_{n\to\infty} m_N(n)^{1/n} = 2(c-2)^{(c-2)/2}(c-1)^{(1-c)}c^{c/2}, \text{ if } c > 2 \text{ and } = 4 \text{ if } c = 2.$$

To prove this, upper and lower bounds for $m_N(n)$ are found, differing byonly $2N$. The author suggests that for large values of N the more appropriate function to consider would be $m_N(n)$ being the smallest integer for which there is a family of sets not having property B, satisfying $A_k \subset S$, $|S| = N$ with the restriction that the set of A_k's contained in any proper subset of S has the property B. A symptotic formulae for $m_N(n)$ and $m'_N(n)$ are not known.

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Classification:
05D05 Extremal set theory
04A99 Miscellaneous topics in set theory