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Set mappings and polarized partition relations (In English)
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[For the entire collection see Zbl 205.00201.]

A set mapping on a set S is a function f from S into the set of subsets of S such that $x \notin f(x)$ $(x \in S)$; $A \subset S$ is called a free set (for the set mapping) if $y \notin f(x)$ for all $x, y \in A$, i.e. $A \cap f(A) = \emptyset$.
In this paper we shall consider set mappings on a well-ordered set S in the case when the order type of S is not necessarily an initial ordinal. In particular, we examine the truth status of the following statement $SM(\alpha, \lambda)$. If f is any set mapping of order α on a set type λ, then there is a free subset having the same order type λ. The Erdős-Specker generalization of the Ruziewicz conjecture asserts that $SM(\alpha, \lambda)$ holds if λ is an infinite initial ordinal and $\alpha < \lambda$. We only examine the problem for the case when $|\lambda| = \aleph_1$ although some of our results hold more generally. We will prove that $SM(\alpha, \lambda)$ holds in the following cases:
(i) $\alpha < \omega_1$ and $\lambda = \omega_1^{\sigma_1+1} + \ldots + \omega_1^{\sigma_k+1} < \omega_1^{\omega+2}$ (k finite);
(ii) $\alpha = \omega_0$ and $\lambda = \omega_1\gamma < \omega_1^{\omega+2}$;
(iii) $\alpha < \omega_0$; $\lambda = \omega\Theta$, where Θ is arbitrary.
Note that the form given for λ in (i) is the most general for which $SM(\alpha, \lambda)$ is true with any $\alpha < \omega_1$.

Classification:
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