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Diagonals of nonnegative matrices. (In English)
Linear multilinear Algebra 1, 89-95 (1973). [0308-1087]

Let $(a_1, \dots, a_n)$, $(r_1, \dots, r_n)$ and $(c_1, \dots, c_n)$ be real n-tuples, $n \geq 3$, satisfying

$$\sum_{i=1}^{n} r_i = \sum_{i=1}^{n} c_i \text{ and } 0 \leq a_i \leq \min(r_i, c_i), \quad i = 1, \dots, n.$$

It is shown that a necessary and sufficient condition for the existence of a nonnegative matrix with main diagonal $(a_1, \dots, a_n)$, with row sums $r_1, \dots, r_n$ and column sums $c_1, \dots, c_n$, is that

$$\sum_{i=1}^{n} (r_i - a_i) \geq \max_t (r_t + c_t - 2a_t).$$

Equality can hold if and only if all the off-diagonal positive entries of the matrix are restricted to the kth row and the kth column, for some k, $1 \leq k \leq n$.

Classification:
15A48 Positive matrices and their generalizations
15A45 Miscellaneous inequalities involving matrices
05B20 (0,1)-matrices (combinatorics)