Zbl 291.10040

Erdős, Paul

On the distribution of numbers of the form σ (n)/n and on some related questions. (In English)

Pac. J. Math. 52, 59-65 (1974). [0030-8730]

An arithmetic function f is said to have a distribution function, if for any c the density $g(c)$ of integers satisfying $f(n) < c$ exists and $g(-\infty) = 0$, $g(\infty) = 1$. Let $f(n) = \sigma(n)/n$, where σ is the sum of divisors function. Then the distribution function g is known to exist, is continuous and monotonic but purely singular. Let $F(x; a, b)$ be the number of integers $n \leq x$ satisfying $a \leq \sigma(n)/n < b$. The author proves the theorem: There is an absolute constant c_1 so that for $x > t$ $F\left(x; a, a + \frac{1}{t}\right) < c_1 x / \ln t$, where apart from the constant c_1 the inequality is best possible. Further from the author's work can be derived some best possible estimates for $g\left(c + \frac{1}{t}\right) - g(t)$ for the case of $\sigma(n)/n$. The author also refers to the relevant problems of abundant numbers and of amicable pairs of numbers. Further he deals with the case where σ is replaced by Euler's φ function and sharpens some earlier known results.

S.M.Kerawala

Classification:
11K65 Arithmetic functions (probabilistic number theory)
11A25 Arithmetic functions, etc.