
Zbl 304.04003

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Splitting almost-disjoint collections of sets into subcollections admitting almost-transversals. (In English)

Infinite finite Sets, Colloq. Honour Paul Erdős, Keszthely 1973, Colloq. Math. Soc. Janos Bolyai 10, 307-322 (1975).

[For the entire collection see Zbl 293.00009.]

Assuming AC, it is proved, given cardinals $n_i \geq 1$ for $i \in I \neq \emptyset$, an integer $m \geq 0$, and ordinals μ, ν, that for the truth of the proposition "every collection of $\aleph_\mu$ sets of cardinal $\aleph_\nu$, any two having $\leq m$ common elements, splits into subcollections $G_i (i \in I)$ each admitting an n_i-transversal: a set S_i with $1 \leq \mathrm{card}(A \cap S_i) < n_i + 1$ for all $A \in G_i$", it is sufficient that either (i) $\mu < \nu$, or (ii) $\mu = \nu + r$ (r finite) and $\sum(n_i + 1) \geq mr + m + 2$, or (iii) $\sum(n_i + 1) \geq \aleph_0$. Some incomplete results are presented supporting the conjecture that the condition is also necessary (assuming GCH), as it is in the case when I is a singleton, due to *P. Erdős* and *A. Hajnal* [Acta Math. Acad. Sci. Hung. 12, 87-123 (1961; Zbl 201.32801)]. The authors do not know whether every collection of sets of $\aleph_1$ different cardinalities, any two having at most one common element, splits into $\aleph_0$ subcollections each admitting a 1- transversal.

Classification:
04A20 Combinatorial set theory
04A25 Axiom of choice and equivalent propositions
04A30 Continuum hypothesis and generalizations
04A10 Ordinal and cardinal numbers; generalizations