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Bollobás, Béla; Erdős, Paul

Alternating Hamiltonian cycles. (In English)

Israel J. Math. 23, 126-131 (1976).

For natural numbers n and d, let $K_n(\Delta_c \leq d)$ denote a complete graph of order n whose edges are colored so that no vertex belongs to more than d edges of the same color, and where Δ_c is the maximal degree in the subgraph formed by the edges of color c. D. E. Daykin proved that if $d = 2$ and $n \geq 6$, then every such graph contains an alternating hamiltonian cycle (i.e. a spanning cycle whose adjacent edges have different colors). The authors have extended this as follows. Theorem: If $69d < n$, then every $G = K_n(\Delta_c \leq d)$ contains an alternating hamiltonian cycle. In fact, it is stated that if $69d < n$, then every $G = K_n(\Delta_c \leq d)$ contains alternating cycles of length ℓ for every ℓ, $3 \leq \ell \leq n$. An analogous result is obtained as follows. Let χ_v denote the number of colors appearing among the edges containing the vertex v, and let $K_n(\chi_v \geq \lambda)$ denote a complete graph of order n whose edges are colored so that each vertex is on at least λ edges of different color. Theorem: Every $K_n(\chi_v \geq (7/8)n)$ contains an alternating hamiltonian cycle. Several related results and conjectures are also presented.

S.F.Kapoor

Classification:
05C35 Extremal problems (graph theory)
05C15 Chromatic theory of graphs and maps