
Zbl 328.10010

Bleicher, Michael N.; Erdős, Paul

Denominators of Egyptian fractions. (In English)

J. Number Theory 8, 157-168 (1976). [0022-314X]

The authors obtain, by elementary methods, good upper and lower bounds for the size of the denominators of Egyptian expansions of fractions and also state several related conjectures. A fraction a/b is said to be written in Egyptian form if we write $a/b = 1/n_1 + 1/n_2 + \ldots + 1/n_k$, $n_1 < n_2 < \ldots < n_k$, where the n_i are positive integers. Let $D(a,b)$ be the minimal value of n_k in all expansions of a/b. Let $D(b)$ be given by $D(b) = \max\{D(a,b) : 0 < a < b\}$. In this work it is shown that $D(b) \leq Kb(lnb)^3$ for some constant K and that for P a prime $D(P) \geq P\{\{\log_2 P\}\}$ where $\{\{x\}\} = -[-x]$ is the least integer not less than x. Both theoretical and computational evidence are given to indicate that $D(N)/N$ is maximum when N is a prime. A number of special cases are dealt with, for example, the authors prove that $D(P^n) < 2P^{n-1}D(P)$. Among the conjectures stated the two of most general interest are, perhaps, (i) $D(N)$ is submultiplicative, i.e., $D(N \cdot M) \leq D(N) \cdot D(M)$. If true, relative primeness of M and N is probably irrelevant. (ii) Let $n_1 < n_2 < \ldots$ be an infinite sequence of positive integers such that $n_{i+1}/n_i > c > 1$. Can the set of rationals a/b for which $a/b = 1/n_{i_1} + 1/n_{i_2} + \ldots + 1/n_{i_t}$ is solvable for some t contain all the rationals in some interval (α, β). We conjecture not. The main results have been improved upon in a second paper by the same authors [Illinois J. Math. 20, 598-613 (1976; Zbl 336.10007.]

Classification:
11A63 Radix representation
11D85 Representation problems of integers