
Zbl 378.04002

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On set systems having paradoxical covering properties. (In English)

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Let ξ be an ordinal, $\varkappa$ a cardinal so that $\xi < \varkappa^+$. A family $B = (B_n : n < \omega)$ of subsets of ξ is said to have the ω-covering property if the union of any ω of these sets is the whole set ξ . On the other hand, the family $B = (B_n : n < \omega)$ is said to be a paradoxical decomposition of ξ if (i) tp. $B_n < \varkappa^n (n < \omega)$ and (ii) B has the ω-covering property. An example of paradoxical decomposition is given from the theorem of Milner and Rado $\xi \twoheadrightarrow (\varkappa^n)^1_{n<\omega}$ if $\xi < \varkappa^+$ The existence of such a partition is related with some results in the theory of polarized partition relations (the authors in Studies pure Math.,63-87 (1971; Zbl 228.04002)). This paper contains a study of $\aleph_2$ phenomena, i.e. of such partition relations whose "next higher case" (i.e. the formula obtained by replacing each cardinal by its successor) is not true. The main reason why it is not possible to extend in a symple way such results is that one of principal tools which were used was the Milner-Rado paradoxical decomposition $\xi \twoheadrightarrow (\varkappa^n)^1_{n<\omega}$ if $\xi < \varkappa$, which higher cardinal analogue is false if we assume $2^{\aleph_1} = \aleph_2$.

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Classification:
04A20 Combinatorial set theory
05A17 Partitions of integres (combinatorics)
03E55 Large cardinals