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Embedding theorems for graphs establishing negative partition relations. (In English)

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The graph G_1 is said to embed into the graph G_1 if G_0 is isomorphic to a spanned subgraph of G_1. Given cardinal numbers $\varkappa$ and λ, the symbol $[\varkappa]$ denotes the complete graph on $\varkappa$ vertices, $[\varkappa\lambda]$ the complete $(\varkappa\lambda)$- bipartite graph and , $[\varkappa/\varkappa]$ the half $(\varkappa/\varkappa)$- bipartite graph (where the set of vertices is a disjoint union $G_0 \cup G_1$ with $|G_0| = |G_1| = \varkappa$ and there are one-to-one enumerations $G_0 = \{x_\alpha; \alpha < \varkappa\}$, $G_1 = \{y_\beta; \beta < \varkappa\}$ such that for each x_α, the set of vertices adjacent to x_α is $\{y_\beta; \alpha < \beta < \varkappa\}$). Let $\triangle_0$, $\triangle_1$ be symbols of these types: The graph G is said to establish the negative partition relation $\varkappa \nrightarrow (\triangle_0, \triangle_1)^2$ if G is a graph on $\varkappa$ vertices such that G contains no subgraph of type $\triangle_0$ and the complement of G contains no subgraph of type $\triangle_1$. The main aim of this paper is to characterize the class of all countable graphs which embed into all graphs G establishing $\aleph_1 \nrightarrow (\triangle_0, \triangle_1)^2$ when $\triangle_0$, $\triangle_1$ are any of $[\aleph_1]$, $[\aleph_1, \aleph_1]$, $[\aleph_1/\aleph_1]$, $[\aleph_0, \aleph_1]$, The authors prove their theorems in ZFC, and then try to show that they are "best possible" assuming the continuum hypothesis or the existence of a Souslin tree. Occasionally Souslin's axiom is invoked instead.

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Classification:
04A20 Combinatorial set theory
03E05 Combinatorial set theory (logic)
03E30 Axiomatics of classical set theory and its fragments
05C99 Graph theory