Zbl 401.10068
Erdős, Paul; Sarközy, A.
On products of integers. II. (In English)
Acta Sci. Math. 40, 243-259 (1978). [0001-6969]

Let k, n be any positive integers. $A = \{a_1, \dots, a_n\}$ any finite, strictly increasing sequence of positive integers satisfying (*) $a_1 = 1$, $a_2 = 2$, $\dots$ $a_k = k$. Let us denote the number of integers which can be written in the form $\prod_{i=1}^{n} a_i^{\varepsilon_i} (\varepsilon_1 = 0$ or $1)$ or $a_i a_j$ $(1 \le i, j \le n)$, respectively by $f(A, n, k)$ and $g(A, n, k)$. Let us write $F(n, k) = \min_A f(A, n, k)$ and $G(n, k) = \min_A g(A, n, k)$, where the minima are extended over all sequence A satisfying (*) and $|A| = n$. The authors conjectured in an earlier paper [Studia Sci. Math. Hung. 9, 161-171 (1974; Zbl 304.10034)] that (1) $G(n, k)/n > c_1$. $G(k, k)/k$ for every $n \ge k$, and furthermore, that for any $\omega > o$, $k > k_0(\omega)$ and $n \ge k$, we have $F(n, k) > n^2 k^{\omega}$ or perhaps (2) $n^2 \exp\left(\frac{c_2 k}{\log k}\right) < F(n, k) < n^2 \exp(c_3 k / \log k)$ for large k and $n \ge k$. In this paper, the authors disprove (1) and prove a slightly weaker form of (2).

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Classification:
11B83 Special sequences of integers and polynomials
11N13 Primes in progressions

Keywords:
products of integers; distribution in sequences