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On the distribution of values of angles determined by coplanar points. (In English)

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The authors study two functions $F(\alpha]$, $G(\alpha]$ which, roughly speaking, denote the ultimate proportion of angles necessarily greater than α, or less than α respectively, in a configuration of coplanar points in general position. The first few sections of the paper are devoted to precise definitions of F, G and showing that the definitions are well- formulated. Also, it is shown that F is a decreasing function on $(0,\pi)$, continuous on the left, while G has opposite behavior. It turns out that $F(\alpha)=1/3$ for $0<\alpha<\pi/3$ and $G(\alpha)=2/3$ for $\pi/2<\alpha<\pi$, but these seem to be the only obvious values of F and G. Various inequalities are proved, namely, $1/9\le F(\pi/2)\le 4/27$, $5/171\le F(2\pi/3)$, $2/8^{1/\varepsilon}\le F(\pi-\varepsilon)$, and $2(\varepsilon/\pi)^3\le G(E)$ for small positive ε, and $1/3\le G(\pi/3)$. Finally, it is shown that $F(\varepsilon+\pi/3)\le 7/27<1/3$, but since $f(\pi/3)=1/3$, F is discontunous (albeit left continuous!) at $\pi/3$. This paper abounds in clever combinatorial arguments of many sorts. Some of the ideas are general enough that they might be put to use in different settings, but useful or not, they are a dilight to read in this paper.

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Classification:
05A99 Classical combinatorial problems
05A20 Combinatorial inequalities

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