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Estimates for sums involving the largest prime factor of an integer and certain related additive functions. (In English)

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Let $P(n)$ denote the largest prime factor of an integer $n \geq 2$ and lrt $\beta(n) = \sum_{p|n} p$, $B(n) = \sum_{p^a||n} p^a$, where as usual $p^a||n$ means that p^a divides n, but p^{a+1} does not. The additive functions β, B and B_1 have been studied recently in several works, including *K.Alladi* and *P.Erdős* [Pac. J. Math. 82, 295-315 (1979; Zbl 419.10042)], *L.-M.DeKoninck* and *A.Ivić* [Topics in arithmetical functions (Amsterdam 1980; Zbl 442.10032)] and *A.Ivić* [Arch. Math. 36, 57-61 (1980; Zbl 436.10019)]. The purpose of this work is to estimate sums of the form $\sum_{2\leq n\leq x} f(n)/g(n)$ when $f \neq g$ and $f, g \in \{P(n), \beta(n), B(n), B_1(n)\}$. Some of these sums have been investigated by the aforementioned authors, and for those which were not following asymptotic formulas are obtained:

$$\sum_{2\leq n\leq x} P(n)/B_1(n) = x + O(x \log\log x/\log x), \tag{1}$$

$$\sum_{2\leq n\leq x} B_1(n)/P(n) = e^{\gamma} x \log\log x + O(x), \tag{2}$$

$$\sum_{2\leq n\leq x} B_1(n)/B(n) = Dx + O(x \log^{-1/3} x), \tag{3}$$

$$\sum_{2\leq n\leq x} B_1(n)/\beta(n) = e^{\gamma} x \log\log x + O(x), \tag{4}$$

Here γ is Eulers's constant, $D > 1$ may be explicitly evaluated, and (1) remains true if $P(n)$ is replaced by either $\beta(n)$ or $B(n)$. The proofs depend on six lemmas and utilize estimates for $\psi(x, y)$, the number of positive integers $\leq x$ all of whose prime factors are $\leq y$. The methods of proof allow one to improve the error term in

$$\sum_{2\leq n\leq x} B(n)/P(n) = x + O(x \log\log x/\log x), \tag{5}$$

to $O(x/\log x)$; (5) was obtained in the aforementioned work of K. Alladi and P. Erdős. The formulas (1)-(4) improve the results of Ch. 6 of the above cited book by J.-M- De Koninck and A. Ivić.

Classification:
11N37 Asymptotic results on arithmetic functions

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sums involving largest prime factor; related additive functions; additive functions; asymptotic formulas