
Zbl 484.10001
Erdős, Paul
Some new problems and results in number theory. (In English)
Number theory, Proc. 3rd Matsci. Conf., Mysore/India 1981, Lect. Notes Math. 938, 50-74 (1982).

[For the entire collection see Zbl 476.00003.]
The problems discussed in this paper are ut into three groups: problems on additive number theory, problems on prime numbers, ans miscellaneous problems. Some of these have been solved, or partially solved, but no solutions are given here. Instead, the author directs our attention to intriguing questions remaining to be answered. He alse continues his custom of offering monetary rewards for solutions to some of the problems. For an example from the first group, let $1 \leq a_1 < \cdots < a_k \leq n$ be asequence of integers for which all the sums $a_1 + a_j$ are distinct. The author and P. Turán have shown [J. Lond. Math. Soc. 16, 212-215 (1941; Zbl 061.07301)] that $\max k = (1 + o(1))n^{1/2}$ and he conjectures that $\max k = n^{1/2} + O(1)$. If the hypothesis weakened so that the number of distinct $a_1 + a_j$ is $(1 + o(1))(k/2)$ it is no longer true that $\max k = (1 + o(1))n^{1/2}$ and an example exists with $k \geq 2n^{1/2}/3^{1/2}$. Hoever, it is conjectured that $\max k =< cn^{1/2}$ for some $c < 2^{1/2}$. Now let $p_1 < p_2 < \dots$ be the sequence of consecutive primes. *R.Rankin* [J. Lond. Math. Soc. 13, 242-247 (1938; Zbl 019.39403)] has shown that fot infinitely many n, and for some $c > 0, p_{n+1} - p_n > cL_n$ where

$$L_n = (\log n)(\log\log n)(\log\log\log\log n)/(\log\log\log n)^2.$$

The author wishes to see a proof that $p_{n+1} - p_n > cL_n$ holds for every c. He has shown [Publ. Math. 1, 33-37 (1949; Zbl 033.16302)] that there is a constant c_1 such that, for infinitely many n, $\min(p_{n+1} - p_n, p_n - p_{n-1}) > c_1 L_n$, and *H.Maier* [Adv. Math. 39, 257-269 (1981; Zbl 457.10023)] has proved that for every k there is a constant c_k such that, for infinitely many n, $\min_{i=1,2,\dots,k}(p_{n+k+1} - p_{n+1} > c_k L_n)$. Nevertheless, it is conjectured that $\lim_{x\to\infty} D_{k+1}(x)/D_k(x) = 0$ where $D_k(x) = \max_{p_n<x} \min_{i=1,\dots,k-1}(p_{n+i+1} - p_{n+i})$. The list of miscellaneous problems begins with the problem of determining whether or not almost all integers have two divisors d_1 and d_2 satisfying $d_1 < d_2 < 2d_1$. The selection here is quite varied.

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Classification:
11-02 Research monographs (number theory)
11B13 Additive bases
11N05 Distribution of primes
00A07 Problem books
11P99 Additive number theory
11B99 Sequences and sets
11M99 Analytic theory of zeta and L-functions

Keywords:

problems on additive number theory; problems on prime numbers; miscellaneous problems; sequence of integers; distinct sums; difference of consecutive primes; divisors of integers

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