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Erdős, Paul; Turk, J.
Products of integers in short intervals. (In English)
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The following properties of distinct integers, say $n_1, \dots, n_f$, from a "short" interval $[n, n+k(n)]$, where $k(n)$ is a "small" function of n (such as $n^{\frac{1}{2}}$, or $\log n$) and $n \geq 1$ is arbitrary, are considered: (1) The product of $n_1, \dots, n_f$ is a perfect power ($\prod_{i=1}^{f} n_i \in \mathbb{N}^m$ for som $m \geq 2$). (2) Two distinct subsets of $\{n_1, \dots, n_f\}$ yield the same product ($\prod_{i \in I_1} n_1 = \prod_{i \in I_2} n_i$). (3) $n_1, \dots, n_f$ are multiplicatively dependent ($\prod_{i \in I_1} n_1^{m_i} = \prod_{i \in I_2} n_i^{m_i}$ for certain $m_i \in \mathbb{N}$). (4) The total number of distinct primes occurring in the prime factorizations of the integers $n_1, \dots, n_f$ is less than the number integers ($\omega(\prod_{i=1}^{f} n_i) < f$). Our results can be summarized as follows: the above properties never occur in "very short" intervals, sometimes in "short" intervals and always in "large" intervals. For example, distinct sets of integers from $[n, n + c_1(\log n)^2(\log\log n)^{-1}]$ have distinct products for any $n \geq 3$, for infinitely many $n \in \mathbb{N}$ this also holds for $[n, n+\exp(c_2(\log n \log\log n)^{\frac{1}{2}})]$, but for infinitely many $n \in \mathbb{N}$ there exists two distinct sets of integers in $[n, n+\exp(c_3(\log n \log\log n)^{\frac{1}{2}})]$, with equal products and for all $n \in \mathbb{N}$ the latter holds for $[n, n + c_4 n^{0.496}]$. The c_1, c_2, c_3, c_4 are absolute positive constants.

P.Erdős

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11D41 Higher degree diophantine equations
11D61 Exponential diophantine equations

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