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On a problem in combinatorial geometry. (In English)

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Let $f(S)$ be the ratio of the area of a largest (nondegenerate) triangle determined by the points of a finite set S to that of a smallest, and $f(n) = \inf_s f(S)$, where the infimum is taken over all planar, noncollinear sets S of cardinality n. It is known that $f(3) = f(4) = 1$, and $f(5) = (\sqrt{5}+1)/2$; it is clear (by taking S_0 to be a set of n points equally spaced and evently distributed on two parallel lines) that $f(n) \leq [\frac{1}{2}(n-1)]$. Using the interesting theorem of E. Sas that the ratio ρ of the area of a convex set C to that of a triangle contained in C having maximal area satisfies the inequality $\rho \leq 4\pi/3\sqrt{3} < 2.4184$, the authors prove that $f(n) = [(n-1)/2]$ for $n > 37$, and that, moreover, for even $n \geq 38$, if $f(S) = f(n)$ then S is affinely equivalent to the set S_0 mentioned above. It is conjectured that $f(n) = [(n-1)/2]$ also for $5 < n \leq 37$, but in this range other extremal configurations besides S_0 are possible. Several other excellent unsolved problems are stated.

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Classification:
52A40 Geometric inequalities, etc. (convex geometry)

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