Zbl 508.05043
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Compactness results in extremal graph theory. (In English)
Combinatorica 2, 275-288 (1982). [0209-9683]

(From the authors' abstract:) "Let L be a given family of ... 'prohibited graphs'. Let $\mathrm{ex}(n, L)$ denote the maximum number of edges a simple graph of order n can have without containing subgraphs from L. A typical extremal graph problem is to determine $\mathrm{ex}(n, L)$, or, at least, to find good bounds on it. Results asserting that, for a given L, there exists a 'much smaller' $L^* \subset L$ for which $\mathrm{ex}(n, L) \approx \mathrm{ex}(n, L^*)$ will be called compactness results. The main purpose of this paper is to prove some compactness results for the case when L consists of cycles. One of our main tools will be finding lower bounds on the number of paths p^{k+1} in a graph on n vertices and E edges ... a 'supersaturated' version of a well known theorem of Erdős and Gallai."
Among the theorems proved, presented in the context of open conjectures, are: Theorem 1: Let k be a natural number. Then $\mathrm{ex}(n, \{C^3, \dots, C^{2k}, C^{2k+1}\}) \leq (n/2)^{1+(1/k)} + 2^k \cdot (n/2)^{1-(1/k)}$. Theorem 2: $\mathrm{ex}(n, \{C^4, C^5\}) = (n/2)^{3/2} + 0(n)$. Theorem 3*: Let T be a tree with a fixed 2-colouring: A graph L is obtained from T by joining a new vertex to each vertex of one colour class by disjoint paths, each k edges long. Then, if $\mathrm{ex}(n, L) \geq cn^{1+(1/k)}$, then is a t for which

$$\lim_{n\to\infty} (ex(n, \{L, C^3, C^5, \dots\})/ex(n, \{L, C^3, C^5, \dots, C^{2t-1}\})) = 1$$

Theorem 5: If $f(n, d)$ is the minimum number of walks W^{k+1} a graph G^n can have with average degree d, then every graph of order n and average degree d contains at least $(1/2) \cdot f(n, d) - 0(f(n, d))$ paths p^{k+1}, as $d \to \infty$.

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Classification:
05C35 Extremal problems (graph theory)
05C38 Paths and cycles
05C15 Chromatic theory of graphs and maps

Keywords:
extremal graph; compactness; supersaturated; disjoint paths