

Zbl 523.10029**Erdős, Paul; Szalay, M.***On some problems of J. Denes and P. Turán.* (In English)**Studies in pure mathematics, Mem. of P. Turán, 187-212 (1983).**

[This article was published in the book announced in Zbl 512.00007.]

Let $\pi = \{\lambda_1 + \lambda_2 + \cdots + \lambda_m = n; \lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_m \geq 1\}$ be a generic partition of n where $m = m(\pi)$ and λ_μ 's are integers. Let $p(n)$ denote the number of partitions of n . J. Denés raised the following problem. What is the number of pairs of partitions of n which do not have equal subsums? This problem has not been solved yet but its investigation led *P. Turán* [Colloq. Math. Soc. János Bolyai 4, 1055-1082 (1969; Zbl 228.05006); Colloq. int. Theorie comb., Roma 1973, Tomo II, 181-200 (1976; Zbl 359.10041); J. Number Theory 6, 405-411 (1974; Zbl 296.05010)] to some unexpected phenomena concerning common summands. Another approach to the original problem would be the investigation of the integers which can be represented by subsums. The following results are proved. Theorem I. The number of partitions of n which represent all integers k of the interval $[1, n]$ as subsums is

$$(1 - \pi(6n)^{-1/2} + o(n^{-1} \log^{30} n)) p(n).$$

The analogue of this assertion does not hold for unequal partitions (their number is $q(n)$) but the following weaker result can be proved. Theorem II. Let K_0 be an integer with $1 \leq k_0 \leq n/2$. Then, the unequal partitions of n represent all integers k of the interval $[k_0, n - k_0]$ as subsums apart from

$$(20(4/3)^{-k_0/2} + o(n^{-1/10}))q(n)$$

unequal partitions of n at most.

Classification:

11P81 Elementary theory of partitions

11P81 Elementary theory of partitions

05A17 Partitions of integres (combinatorics)

Keywords:

statistical properties of partitions; unequal partitions of integers; partition-subsums