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On the statistical theory of partitions. (In English)

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[For the entire collection see Zbl 541.00002.]

Let $\Pi = \{\lambda_1 + \lambda_2 + ... + \lambda_m = n; \quad \lambda_1 \geq \lambda_2 \geq ... \geq \lambda_m \geq 1\}$ be a generic partition of n where $m = m(\Pi)$ and the $\lambda'_\mu s$ are integers. Let $p(n)$ denote the number of partitions of n. The first author and *J. Lehner* [Duke Math. J. 8, 335-345 (1941; Zbl 025.10703)] determined the distribution of λ_1 where $\lambda_1 = \lambda_1(\Pi) = \max_{\nu\in\Pi}\nu$. The following analogous result is proved for the maximum with multiplicities. Theorem 1. The number of partitions of n with the property

$$\max_{\nu\in\Pi}\{\nu \text{mult}(\nu) \text{ in } \Pi\} \leq (2\pi)^{-1}(6n)^{1/2}\log n + \pi^{-1}(6n)^{1/2}\log\log\log n + \pi^{-1}(6n)^{1/2}c$$

is $(\exp(-\pi^{-1}6^{1/2}e^{-c}) + o(1))p(n)$.

As to the λ_μ's, some consequences of earlier results are also discussed. For "unequal" partitions (their number is $q(n)$), the increasing order $(\alpha'_1 + ... + \alpha'_m = n; 1 \leq \alpha'_1 < \alpha'_2 < ... < \alpha'_m)$ is more interesting. Theorems 2 and 3 state estimates for α'_μ which yield the following Corollary. For arbitrary $\eta > 0$, there exist n_0 and $\epsilon > 0$ such that, for $n > n_0$ with the restriction $\epsilon^{-1} \leq \mu \leq \epsilon \cdot n^{1/2}$, the estimation $|\alpha'_\mu - 2\mu| \leq \eta\mu$ holds uniformly with the exception of at most $\eta q(n)$ unequal partitions of n.

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