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The closed linear span of $\{x^k - c_k\}_1^\infty$. (In English)

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Several easily verified conditions on a sequence $(c_k)_1^\infty$ of real numbers are given which imply that the sequence of functions $(x^k - c_k)_1^\infty$ is total in $C[0,1]$. This problem is equivalent to demanding that the function $f(x) \equiv 1$ belongs to the closed linear hull of $(x^k - c_k)_1^\infty$ in $C[0,1]$. For instance, if the sequence $(c_k)_1^\infty$ is such that for all $k \geq M$, $\epsilon(-1)^k(c_k - c) \geq 0$, where $c \in \mathbb{R}$ and $\epsilon \in \{-1, 1\}$, fixed, and if $c_k - c \not\equiv 0$, then $(x^k - c_k)_1^\infty$ is total in $C[0,1]$; if, in addition, $c_k \neq c$ for infinitely many k, with the help of Chebyshev polynomials an effective approximation to $f(x) \equiv 1$ in $C[0,1]$ by finite linear combinations of the $x^k - c_k$ is given. Another condition is: $|c_{n_k} - c|^{1/n_k} \to 0$ as $k \to \infty$, where the subsequence $(n_k)_1^\infty$ satisfies the Müntz condition $\sum_{k=1}^\infty (n_k)^{-1} = \infty$ and $c_k \not\equiv c$; in the case when $|c_k|^{1/k} \to 0$ as $k \to \infty$, again, a good approximation to f(x)$\equiv$ 1 is explicitly constructed.

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Chebyshev polynomials; Müntz condition