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Maximum induced trees in graphs. (In English)

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The paper studies $t(G)$ = maximum size of a subset of vertices of a graph that induces a tree. Upper and lower bounds are established in terms of other invariants of G. There is a lower bound in terms of the radius: $t(G) \geq 2\mathrm{rad}(G)-1$. With α being the independence number and $1 \leq m \leq (n-1)/2$ holds: $\alpha(G) > ((m-1)n)/m+1$ implies $t(G) \geq 2m+1$ and $\alpha(G) > ((m-1)n+1)/m+1$ implies $t(G) \geq 2m+2$, these bounds being best possible ($m = |E(G)|$, $n = |V(G)|$). Let $f(n,\rho)$ = minimum of $t(G)$ over all graphs G with n vertices and $n+\rho-1$ edges. Upper and lower bounds for $f(n,\rho)$ are obtained resulting in an almost complete description of the asymptotic behavior of $f(n,\rho)$. This shows that $f(n,\rho)$ is of a surprisingly small order. Relations between $t(G)$ and the maximum clique size are proved: For $k \geq 3$, $t \geq 2$ there is a minimum integer $N(k,t)$ such that every connected graph with at least $N(k,t)$ vertices has either a clique of size k or an induced tree of size t. For $N(k,t)$ bounds are derived. Finally, the problem "For given G and t is $t(G) > t?''$ is NP complete.

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Classification:
05C35 Extremal problems (graph theory)
05C05 Trees

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