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Erdős, Paul; Nathanson, Melvyn B.
Independence of solution sets in additive number theory. (In English)
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Let $A \subseteq \mathbb{N}$, then A is called an asymptotic basis of order 2 if for all sufficiently large $n \in \mathbb{N}$ there are $a, a' \in A$ such that $n = a+a'$. Let $S_A(n) = \{a \in A|\ n-a \in A,\ n \neq 2a\}$ denote the solution set of n. By the Erdős-Rényi probabilistic method [see *H.Halberstam* and *K.F.Roth*, Sequences (1966; Zbl 141.04405), p. 141 ff.] it is shown that for almost all A in the space Ω of all strictly increasing sequences of positive integers the cardinality of $S_A(m) \cap S_A(n)$ is bounded for all $m < n$. The bound depends on the chosen probability measure on Ω only. This result is useful to proof the existence of minimal asymptotic bases A of order 2, which means A has no proper subset being an asymptotic basis of order 2 itself. It is proved that $A \subseteq \mathbb{N}$ contains a minimal asymptotic basis of order 2 if $|S_A(m) \cap S_A(n)|$ is bounded for all $m < n$ and $\lim_{n\to\infty} |S_A(n)| = \infty$.

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