
Zbl 623.01010
Erdős, Paul
My joint work with Richard Rado. (In English)
Surveys in combinatorics 1987, Pap. 11th Br. Combin. Conf., London/Engl. 1987, Lond. Math. Soc. Lect. Note Ser. 123, 53-80 (1987).
[For the entire collection see Zbl 611.00003.]
This is a long but entertaining article describing the joint work of Erdős and Rado (and others such as A. Hajnal). Many solved and unsolved problems are discussed and various amounts of money are offered for the solutions of various problems. The problems are clearly stated. For example the first collaborative work was on the following problem: Let

$$|S| = n, \quad n \geq K, \quad A_i \subset S, \quad 1 \leq i \leq t(n:K)$$

be an intersecting family of subsets of S (i.e. $A_i \cap A_j \neq \emptyset$) such that $|A_i| = K$ then

$$t(n:K) \leq \binom{n-1}{K-1}.$$

Most of the latter part of the article is based on problems involving the partition symbol introduced by Rado and Erdős

$$A \to (B_n)^r_{h \in H}$$

which, as Erdős would say, 'in human language' means that if we divide the r-tuples of A (which is a cardinal, ordinal or order type) into H classes (H a set) then for some $h \in H$ there is a subset of type B_n such that all r-tuples of B_n are in the same class. The equivalent symbol with $\nrightarrow$ replacing $\to$ means this is not true. Two examples given are:

$$\aleph_0 \to (\aleph_0)^r_K \quad (K, r < \aleph_0)$$

(that is if we split the r-tuples of a denumerable set into K classes there is always an infinte set all of whose r-tuples are in the same class) and

$$C \nrightarrow (\aleph_1, \aleph_1)^2_2$$

(that is one can partition the pairs of real numbers into two classes such that every subset of power $\aleph_1$ contains a pair from both classes). Many such problems are considered.

B.Burrows

Classification:
01A60 Mathematics in the 20th century
06-03 Historical (ordered structures)
04-03 Historical (set theory)
05-03 Historical (combinatorics)
00A07 Problem books

Keywords:
denumerable set; partition